6.2: The number of elements is a finite set

Counting problem: find the number of elements in a set.

EXAMPLE 1. *If*

$$
\begin{aligned}
U &= \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} \\
A &= \{1, 2, 3, 4, 5\} \\
B &= \{-1, -2, -3, -4, -5\}
\end{aligned}
$$

then

- $n(U) =$
- $n(B) =$
- $n(A \cap B) =$

- $n(A) =$
- $n(A \cup B) =$
- $(n(A \cup B)^c) =$

EXAMPLE 2. *If $n(U) = 135$, $n(A) = 55$, $n(B) = 45$, and $n(A \cap B) = 20$, how many elements are in $A \cup B$?*

EXAMPLE 3. *If $n(U) = 135$, $n(A) = 55$, $n(B) = 45$, and $n(A \cap B) = 0$, how many elements are in $A \cup B$?*

Union formulas:

$$
n(A \cup B) = n(A) + n(B) - n(A \cap B)
$$

In particular, if A and B are disjoint (i.e. $A \cap B = \emptyset$ (have nothing in common)), then

$$
n(A \cup B) = n(A) + n(B)
$$

because $n(A \cap B) = 0$ in this case.

EXAMPLE 4. *Of* 50 *employees of a store located in downtown Boston,* 18 *people take the subway to work,* 12 *take the bus, and* 7 *take both the subway and the bus. How many employees*

(a) *Take the subway or the bus to work?*

(b) *Take only bus to work?*

(c) *Take either the bus or the subway to work?*

(d) *Get to work by some other means?*

EXAMPLE 5. **(a)** *Completely fill in the Venn diagram below given the following information:*

$$n(U) = 100, \quad n(A) = 28, \quad n(B) = 30, \quad n(C) = 34,$$

$$n(A \cap B) = 8, \quad n(A \cap C) = 10, \quad n(B \cap C) = 15,$$

$$n(A \cap B \cap C) = 5.$$

U

(b) *Compute* $n\left[A \cap (B \cup C)\right]$

**(c)** *Compute* $n\left[A \cap (B \cup C)^c\right]$

EXAMPLE 6. *A survey of* 300 *high school seniors revealed that*

    120 *students had not read Macbeth but had read Hamlet or Romeo and Juliet;*

    61 *had read Hamlet but not Romeo and Juliet;*

    15 *had read Macbeth and Hamlet;*

    14 *had read Hamlet and Romeo and Juliet;*

    9 *had read Macbeth and Romeo and Juliet;*

    5 *had read Macbeth and Romeo and Juliet but not Hamlet;*

    40 *had read only Macbeth.*

    Let $M =$ Macbeth, $H =$ Hamlet *and* $R =$ Romeo and Juliet;

**(a)** *Fill in a Venn diagram illustrating the above information:*

U
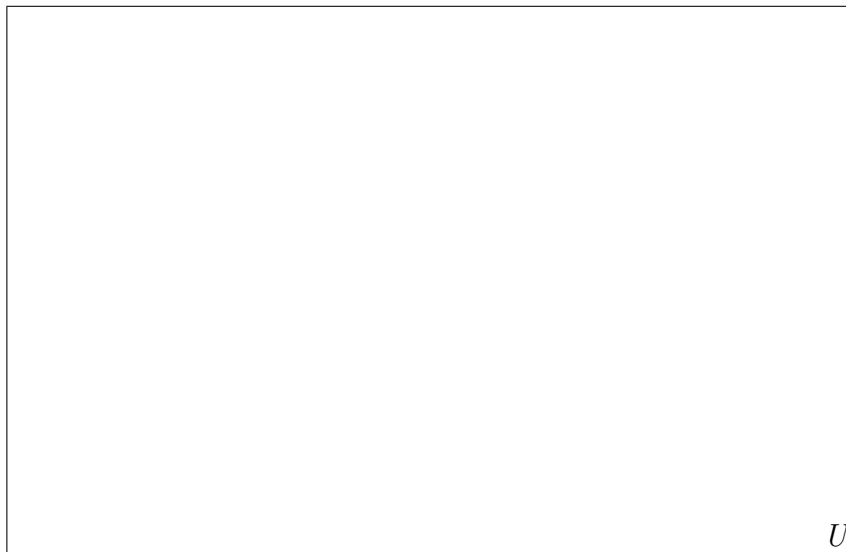
(b) *How many students read exactly one of these books?*

(c) *How many students did not read* Romeo and Juliet*?*

(d) *How many students read* Macbeth *or* Hamlet *and also read* Romeo and Juliet*?*

(e) *Compute* $n\left[M \cup (R^c \cap H)\right] =$

EXAMPLE 7. *In a survey of 300 individual investors regarding subscriptions to the* **New York Times** *(NYT),* **Wall Street Journal** *(WSJ), and* **USA Today** *(UST), the following data were obtained:*

- *122 subscribe to NYT*

- *150 subscribe to WSJ*

- *62 subscribe to UST*

- *38 subscribe to NYT and WSJ*

- *20 subscribe to NYT and UST*

- *28 subscribe to WSJ and UST*

- *36 do not subscribe to any of these newspapers.*

(a) *Fill in a Venn diagram illustrating the above information:*
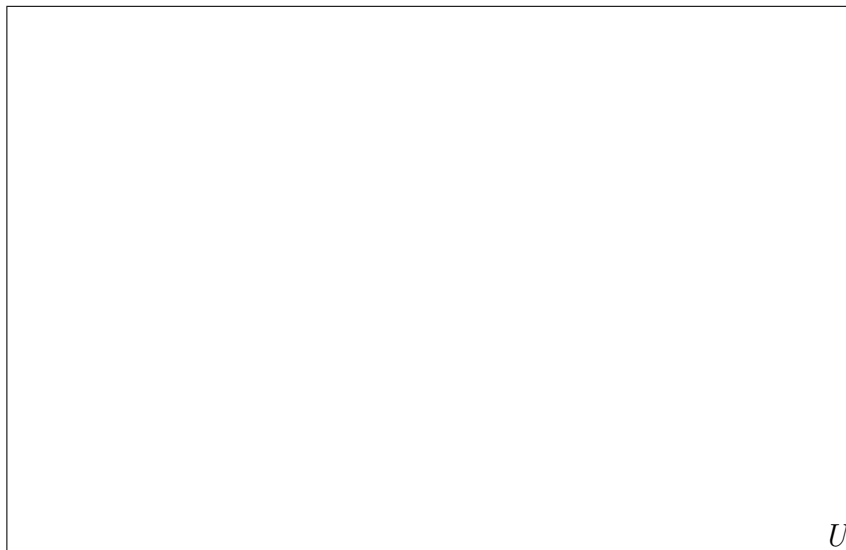
(b) *How many of the individual investors surveyed subscribe to all three newspapers?*

(c) *How many subscribe to only one of these newpapers?*