6.2: The number of elements is a finite set

$n(*) = \#$ elements in set $*$
$n(A) = \#$ elements in A

Counting problem: find the number of elements in a set.

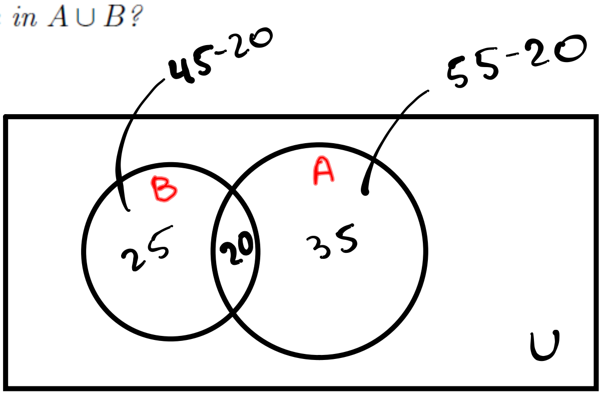
EXAMPLE 1. *If*

$$\begin{aligned} U &= \{-5,-4,-3,-2,-1,0,1,2,3,4,5\} \\ A &= \{1,2,3,4,5\} \\ B &= \{-1,-2,-3,-4,-5\} \end{aligned}$$

then

- $n(U) = 11$
- $n(A) = 5$
- $n(B) = 5$
- $n(A \cup B) = 10$
- $n(A \cap B) = n(\emptyset) = 0$
- $(n(A \cup B)^c) = n(\{0\}) = 1$

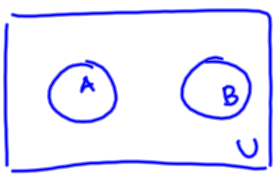
EXAMPLE 2. *If* $n(U) = 135$, $n(A) = 55$, $n(B) = 45$, *and* $n(A \cap B) = 20$, *how many elements are in* $A \cup B$?



$n(A \cup B) = \underbrace{25 + 20}_{n(B)} + \underbrace{35}_{n(A) - n(A \cap B)} = \boxed{80}$

EXAMPLE 3. *If* $n(U) = 135$, $n(A) = 55$, $n(B) = 45$, *and* $\underline{n(A \cap B) = 0}$, *how many elements are in* $A \cup B$?

A & B are disjoint



$n(A \cup B) = n(A) + n(B) =$
$= 55 + 45 = \boxed{100}$

Union formulas:

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

In particular, if A and B are disjoint (i.e. $A \cap B = \emptyset$ (have nothing in common)), then

$$n(A \cup B) = n(A) + n(B)$$

because $n(A \cap B) = 0$ in this case.

Remark: $\boxed{n(A \cap B) = n(A) + n(B) - n(A \cup B)}$

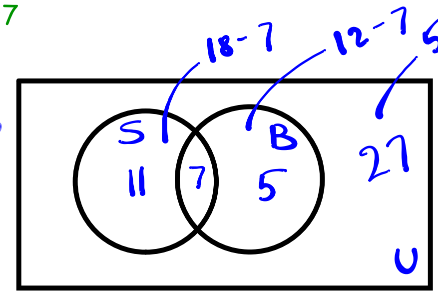
EXAMPLE 4. *Of 50 employees of a store located in downtown Boston, 18 people take the subway to work, 12 take the bus, and 7 take both the subway and the bus. How many employees*

(a) *Take the subway or the bus to work?* (S ∪ B)

$n(S \cup B) = n(S) + n(B) - n(S \cap B) = 18 + 12 - 7 = \boxed{23 \text{ empl.}}$

U={x|x is empl. of store}, n(U)=50
S={x|x takes subway}, n(S)=18
B={x|x takes bus}, n(B)=12
n(S∩B)=7

(b) *Take only bus to work?* $\boxed{5 \text{ empl.}}$ →



(c) *Take either the bus or the subway to work?*

$11 + 5 = \boxed{16 \text{ empl.}}$

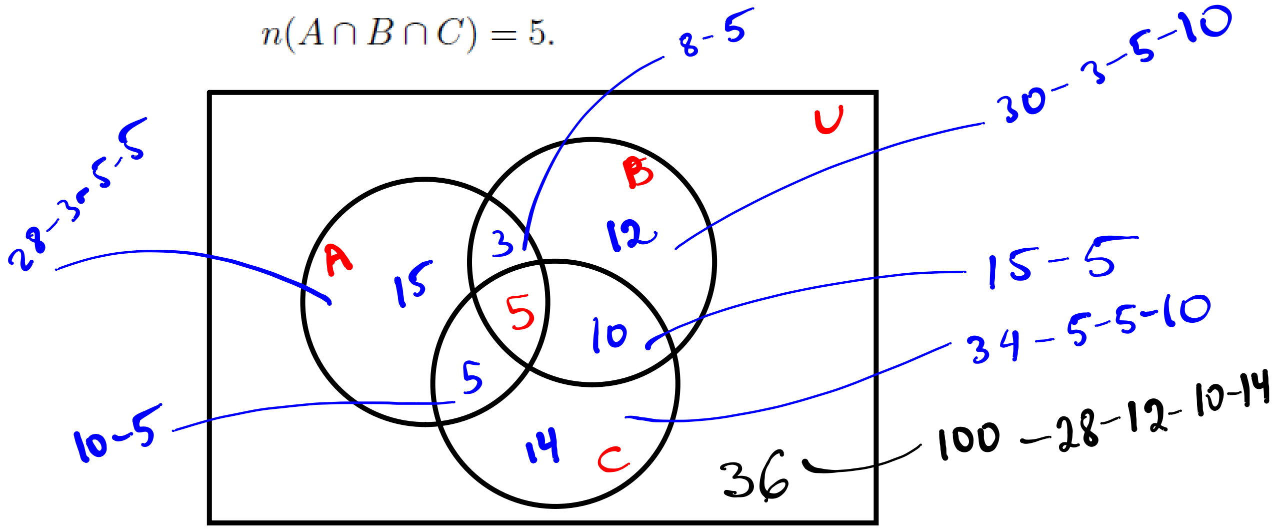
(d) *Get to work by some other means?* $\boxed{27 \text{ empl.}}$

EXAMPLE 5. (a) *Completely fill in the Venn diagram below given the following information:*

$$n(U) = 100, \quad n(A) = 28, \quad n(B) = 30, \quad n(C) = 34,$$

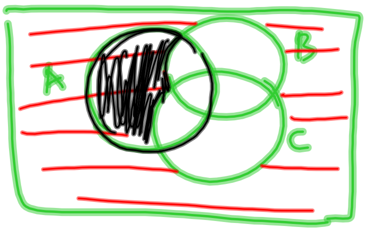
$$n(A \cap B) = 8, \quad n(A \cap C) = 10, \quad n(B \cap C) = 15,$$

$$n(A \cap B \cap C) = 5.$$



(b) *Compute* $n[A \cap (B \cup C)]$ $= 5+5+3$

(c) *Compute* $n[A \cap (B \cup C)^c]$ $= 15$



$A \cap (B \cup C)^c =$

$= \{x \mid x \in A \text{ and } x \notin B \cup C\}$

EXAMPLE 6. *A survey of 300 high school seniors revealed that* $n(U) = 300$

120 *students had not read Macbeth but had read Hamlet or Romeo and Juliet;* $M^c \cap (H \cup R)$

61 *had read Hamlet but not Romeo and Juliet;*

15 *had read Macbeth and Hamlet;*

14 *had read Hamlet and Romeo and Juliet;*

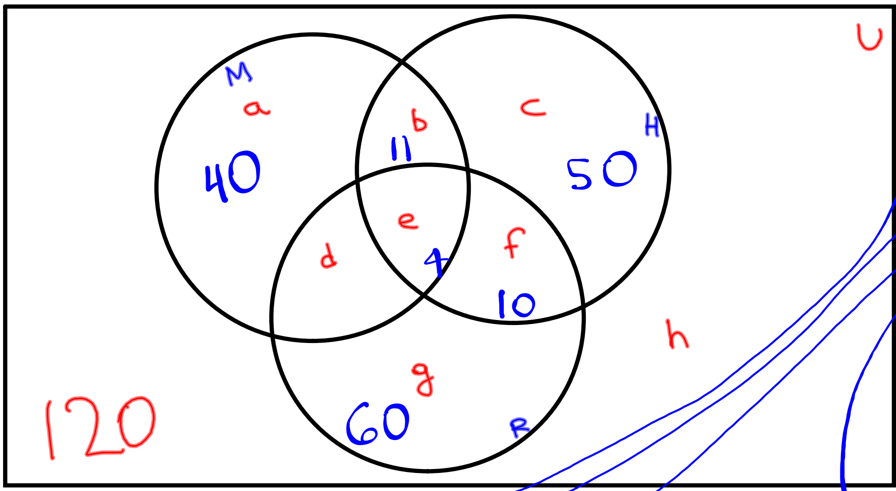
9 *had read Macbeth and Romeo and Juliet;*

5 *had read Macbeth and Romeo and Juliet but not Hamlet;*

40 *had read only Macbeth.*

Let $M =$ read Macbeth, $H =$ read Hamlet *and* $R =$ read Romeo and Juliet*;*

(a) *Fill in a Venn diagram illustrating the above information:*



$a+b+c+d+e+f+g+h = 300$

$c+f+g = 120$

$b+c = 61$

$b+e = 15$

$e+f = 14$

$d+e = 9$

$d = 5$

$a = 40$

$e = 9-d = 9-5 = 4$

$f = 14-e = 14-4 = 10$

$c = 61-b = 61-11 = 50$ $\quad b = 15-e = 15-4 = 11$

$g = 120-c-f = 120-50-10 = 60$

$h = 300-a-b-c-d-e-f-g = 300-40-11-50-5-4-10-60 = 120$

(b) *How many students read exactly one of these books?*

$a+c+g = 40+60+50 = \boxed{150 \text{ students}}$

(c) *How many students did not read Romeo and Juliet?*

(d) *How many students read Macbeth or Hamlet and also read Romeo and Juliet?*

$n(R^c) = n(U-R) = a+b+c+h = 40+11+50+120 = \boxed{221}$

(e) *Compute* $n[(M \cup H) \cap (R^c \cap H)]$.

$n((M \cup H) \cap R) = d+e+f = 5+4+10 = 19$

$a+b+d+e+c = 40+5+4+11+50 = \boxed{110}$

$M = \{a, b, d, e\}$

$R^c = \{a, b, c, h\}$ $\Rightarrow R^c \cap H = \{b, c\}$ $\Rightarrow$

$H = \{b, c, e, f\}$

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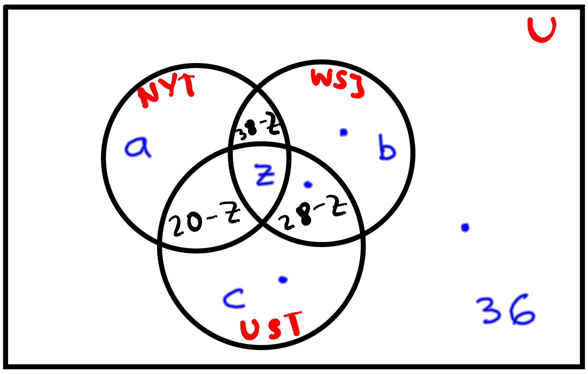
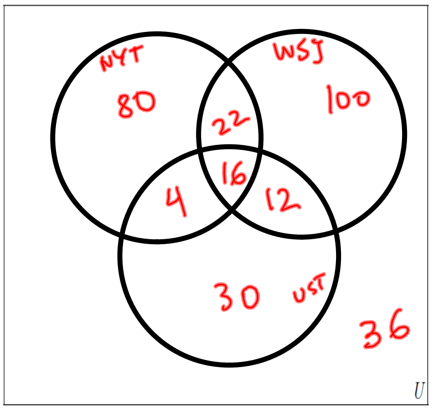
EXAMPLE 7. *In a survey of 300 individual investors regarding subscriptions to the New York Times (NYT), Wall Street Journal (WSJ), and USA Today (UST), the following data were obtained*

- *122 subscribe to NYT*
- *150 subscribe to WSJ*
- *62 subscribe to UST*
- *38 subscribe to NYT and WSJ*
- *20 subscribe to NYT and UST*
- *28 subscribe to WSJ and UST*
- *36 do not subscribe to any of these newspapers.*



$$62 = c + 20 - \cancel{z} + \cancel{z} + 28 - z$$
$$150 = b + 38 - \cancel{z} + \cancel{z} + 28 - z$$
$$122 = a + 38 - \cancel{z} + \cancel{z} + 20 - z$$
$$\cancel{300 = a + b + c + 36 + 38 - z + z + 20 - z + 28 - z}$$
$$36 + 122 + b + 28 - z + c = 300$$

(a) *Fill in a Venn diagram illustrating the above information:*



simplify

$$c - z = 14$$
$$b - z = 84$$
$$a - z = 64$$
$$b + c - z = 114$$

$$\begin{array}{c} \begin{array}{cccc} a & b & c & z \end{array} \\ \left[\begin{array}{cccc|c} 0 & 0 & 1 & -1 & 14 \\ 0 & 1 & 0 & -1 & 84 \\ 1 & 0 & 0 & -1 & 64 \\ 0 & 1 & 1 & -1 & 114 \end{array}\right] \end{array}$$

$\downarrow$ RREF

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 80 \\ 0 & 1 & 0 & 0 & 100 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 16 \end{array}\right] \quad \begin{array}{l} a = 80 \\ b = 100 \\ c = 30 \\ z = 16 \end{array}$$

(b) *How many of the individual investors surveyed subscribe to all three newspapers?* $\boxed{16}$

(c) *How many subscribe to only one of these newpapers?* $80 + 100 + 30 = \boxed{210}$