6.4: Permutation and Combinations

EXAMPLE 1. *How many ways can you arrange* 10 *books on a shelf together?*

Given a set of distinct objects, a permutation of a set is an arrangement of these objects in a definite order. Therefore, *the number of permutations of* n *distinct objects taken* n *at a time is* $n!$.

DEFINITION 2. *The number of permutations,* $P(n,r)$*, of* n *distinct items of which* r *objects are chosen to be placed in an* <u>*ordered*</u> *setting (i.e. row, list,...) is given by*

$$P(n,r) = \frac{n!}{(n-r)!}$$

On your calculator: $\boxed{\text{MATH}}$ → PRB → #2nPr.

EXAMPLE 3. *Compute*

$P(n,n) =$ $\qquad$ $P(n,1) =$ $\qquad$ $P(7,4) =$

EXAMPLE 4. *Find the number of ways a chairman, a vice-chairman, and a secretary can be chosen from a committee of eleven members.*

DEFINITION 5. *The number of combinations,* $C(n,r)$*, of* n *distinct items of which* r *objects are chosen to be placed in an* <u>*unordered*</u> *setting is given by*

$$C(n,r) = \frac{n!}{(n-r)!r!}, \quad \text{where } r \leq n.$$

On your calculator: $\boxed{\text{MATH}}$ → PRB → #3nCr.

EXAMPLE 6. *A subcommittee of three members is to be selected from a committee of eleven members. Determine the number of ways this can be done.*

EXAMPLE 7. *How many ways can* 2 *subcommittees be formed where one has* 4 *people and the other has* 3 *people from a committee of eleven members.*

EXAMPLE 8. **(a)** *How many different ways can* 4 *books be selected from a pile of* 10 *different books and arranged on a shelf?*

(b) *How many ways can you select* 4 *books to read next week from a pile of* 10 *different books?*

EXAMPLE 9. *Six friends go to the movies and all sit in the first row, which has* 10 *seats available. How many different seating arrangements of these six friends are possible in these* 10 *seats?*

EXAMPLE 10. *Lauren has a bucket of sidewalk chalk. In the bucket there are* 2 *green,* 8 *red,* 2 *yellow,* 4 *blue and* 4 *pieces of white chalk. If she randomly pulls out* 6 *pieces of chalk, in how many ways can she pull out exactly* 2 *red chalks and* 1 *white chalk?*

EXAMPLE 11. *A box contains* 4 *lime,* 8 *cherry and* 10 *blue raspberry Jolly Ranchers. If Jessica randomly selects* 7 *Jolly Ranchers from the box, in how many ways could she select exactly* 5 *of the same color?*

EXAMPLE 12. *A box contains 800 DVD's of which* 50 *are scratched. In how many ways can you randomly select* 25 *DVD's such that at least* 2 *are scratched?*

EXAMPLE 13. *Six cards are randomly selected from a standard deck of* 52 *playing cards. How many* 6*-card hands are possible*

(a) *if there are no restrictions?*

(b) *that have exactly* 4 *hearts or exactly* 3 *spades cards?*

Distinct rearrangements (or, permutations of n objects, Not All Distinct):

EXAMPLE 14. *How many distinguishable ways can you rearrange the letters in the word BEAR? What about the word BEER?*

BEAR:

BEAR	*BERA*	*BARE*	*BAER*	*BREA*	*BRAE*
EBAR	*EBRA*	*EABR*	*EARB*	*ERBA*	*ERAB*
ABRE	*ABER*	*ARBE*	*AREB*	*AEBR*	*AERB*
RBEA	*RBAE*	*REBA*	*REAB*	*RABE*	*RAEB*

BEER:

BEER	*BERE*	*BERE*	*BEER*	*BREE*	*BREE*
EBER	*EBRE*	*EEBR*	*EERB*	*ERBE*	*EREB*
EBRE	*EBER*	*ERBE*	*EREB*	*EEBR*	*EERB*
RBEE	*RBEE*	*REBE*	*REEB*	*REBE*	*REEB*

•*If we have n objects in which n_1 of the objects are alike (same), then the number of permutations of these n objects taken n at a time would be* $\dfrac{n!}{n_1!}$.

EXAMPLE 15. *How many different arrangements can be made from the letters of MASSACHUSETTS?*

Appendix: **Standard Deck of Cards:** A deck of cards has 4 suits: *diamonds, hearts, clubs, and spades.* The suits of diamonds and hearts are both *red* and the suits of clubs and spades are both **black**. Each suit has the following denominations: Ace, 2, 3, 4, 5, 6, 7, 8, 9,10, Jack, Queen, and King. The Jacks, Queens and Kings are also called *face cards.*