

7.2: Definition of Probability

The **probability** of an event E happening is the number of ways that specific event can happen divided by the number of possible outcomes in the whole experiment:

$$P(E) = \frac{n(E)}{n(S)}.$$

EXAMPLE 1. *An experiment consists of tossing a fair coin one time and observing if it lands on heads or tails. Find the probability of tossing a head and the probability of tossing a tail.*

The probability of an event is a number that is between 0 and 1, inclusive. The larger the number, the more likely the event will occur.

DEFINITION 2. *A sample space where every outcome has the same probability (chance of happening) is called a **uniform sample space** and $P(\text{any individual outcome}) =$*

DEFINITION 3. *An empirical **probability distribution** is a chart that shows the probability of every single outcome (simple event) in the sample space.*

Outcome	s_1	s_2	s_3	\dots	s_n
Probability	$P(s_1)$	$P(s_2)$	$P(s_3)$	\dots	$P(s_n)$

Note that $P(s_1) + P(s_2) + P(s_3) + \dots + P(s_n) = 1$.

EXAMPLE 4. *An experiment consists of rolling a fair die one time and observing the number landing uppermost. Determine the sample space, simple events, and find the probability distribution for this experiment.*

EXAMPLE 5. A group of people living with their children under the age of 18 was asked how many days per week they cook at home. The results of the survey are summarized below. Determine the (empirical) probability distribution associated with this survey.

# Days	0	1	2	3	4	5	6	7
# Respondents	25	30	45	75	55	100	85	85

FINDING THE PROBABILITY OF AN EVENT

1. Determine a sample space S associated with the experiment.
2. Assign probabilities to the simple events of S .
3. If $E = \{s_1, s_2, \dots, s_n\}$ where $\{s_1\}, \{s_2\}, \dots, \{s_n\}$ are simple events then

$$P(E) = P(\{s_1\}) + P(\{s_2\}) + \dots + P(\{s_n\}).$$

If E is the empty set, \emptyset , then $P(E) = 0$.

EXAMPLE 6. The number of grades for a group of Math 141 students are shown in the table:

Grade	A	B	C	D	F
Frequency	30	46	10	3	9

If a student from this group is selected at random, what is the probability that the student made

(a) an A?

(b) an A or a B?

EXAMPLE 7. Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the sample space associated with an experiment having the following probability distribution:

Outcome	s_1	s_2	s_3	s_4	s_5
Probability	$\frac{4}{15}$		$\frac{7}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

Let $A = \{s_1, s_2, s_5\}$, $B = \{s_2, s_4, s_5\}$, $C = S$. Compute:

(a) $P(C) =$

(b) $P(\emptyset) =$

(c) $P(\{s_2\}) =$

(d) $P(A) =$

(e) $P(B) =$

(f) $P(B^c)$

(g) $P(A \cap B) =$

(h) $P(A \cup B) =$

EXAMPLE 8. A pair of fair dice are rolled one time. Sample space for this experiment is below.

$$S = \{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \}$$

(a) Calculate the probability that the two dice show the same number.

(b) Calculate the probability that the sum of the two die is 6.

(c) Give the probability distribution for the sum of the die.

(d) Calculate the probability that the sum of the two die is a multiple of 5.

EXAMPLE 9. *If a card is drawn at random from a standard 52-card deck, what is the probability that the card drawn is*

(a) *a diamond*

(b) *a red card*

(c) *a king*

EXAMPLE 10. *A ball is selected at random from an urn containing 3 yellow, 4 red, and 5 green balls. What is the probability that:*

(a) *the ball is green?*

(b) *the ball is green or red?*

(c) *the ball is purple?*

(c) *the ball is yellow, green or red?*