7.2: Definition of Probability

The **probability** of an event E happening is the number of ways that specific event can happen divided by the number of possible outcomes in the whole experiment:

$$P(E) = \frac{n(E)}{n(S)}. \qquad 0 \leq P(E) \leq 1$$

EXAMPLE 1. *An experiment consists of tossing a fair coin one time and observing if it lands on heads or tails. Find the probability of tossing a head and the probability of tossing a tail.*

H T E F

$S = \{T, H\}$

outcomes simple events $\begin{cases} E = \{H\} \\ F = \{T\} \end{cases} \Longrightarrow$

$P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{1}{2}}$

$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{1}{2}}$

probability distribution →

Outcomes	{H}	{T}
Probability	$\frac{1}{2}$	$\frac{1}{2}$

$= 1$

$P(\{H\}) = P(\{T\}) \Rightarrow S$ is uniform sample space

The probability of an event is a number that is between 0 and 1, inclusive. The larger the number, the more likely the event will occur.

$0 \leq P(E) \leq 1$

$P(\varnothing) = \frac{n(\varnothing)}{n(S)} = \frac{0}{n(S)} = 0$

$P(S) = \frac{n(S)}{n(S)} = 1$

DEFINITION 2. *A sample space where every outcome has the same probability (chance of happening) is called a* **uniform sample space** *and* $P(\textit{any individual outcome}) = \frac{1}{n(S)}$

DEFINITION 3. *An empirical* **probability distribution** *is a chart that shows the probability of every single outcome (simple event) in the sample space.*

Outcome	$\{s_1\}$	$\{s_2\}$	$\{s_3\}$	$\dots$	$\{s_n\}$
Probability	$P(\{s_1\})$	$P(\{s_2\})$	$P(\{s_3\})$	$\dots$	$P(\{s_n\})$

$= 1$

Note that $P(s_1) + P(s_2) + P(s_3) + \cdots + P(s_n) = 1$.

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EXAMPLE 4. *An experiment consists of rolling a fair die one time and observing the number landing uppermost. Determine the sample space, simple events, and find the probability distribution for this experiment.*

6-sided

Sample space: $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Simple events: E: $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$

Probability distribution:

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$$

Outcome	$\{1\}$	$\{2\}$	$\{3\}$	$\{4\}$	$\{5\}$	$\{6\}$
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Remark: S is uniform

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EXAMPLE 5. *A group of people living with their children under the age of* 18 *was asked how many days per week* (outcome) *they cook at home. The results of the survey are summarized below. Determine the (empirical) probability distribution associated with this survey.*

Outcome	*# Days*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	
Frequency	*# Respondents*	*25* +	*30* +	*45* +	*75* +	*55* +	*100* +	*85* +	*85*	= 500 total #

Probability=relative frequency=outcome frequency/total #

Outcome, # days	0	1	2	3	4	5	6	7
Relative frequency= probability	$\frac{25}{500}$	$\frac{30}{500}$	$\frac{45}{500}$	$\frac{75}{500}$	$\frac{55}{500}$	$\frac{100}{500}$	$\frac{85}{500}$	$\frac{85}{500}$

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FINDING THE PROBABILITY OF AN EVENT

1. Determine a sample space S associated with the experiment. = outcomes
2. Assign probabilities to the simple events of S. = prob. distribution
3. If $E = \{s_1, s_2, \ldots, s_n\}$ where $\{s_1\}, \{s_2\}, \ldots, \{s_n\}$ are simple events then

$$\boxed{P(E) = P(\{s_1\}) + P(\{s_2\}) + \ldots + P(\{s_n\}).}$$

If E is the empty set, $\emptyset$, then $P(E) = 0$.

EXAMPLE 6. *The number of grades for a group of Math 141 students are shown in the table:*

Grade, outcome	*A*	*B*	*C*	*D*	*F*
Frequency	*30*	*46*	*10*	*3*	*9*

= 98

$S = \{A, B, C, D, F\}$

Outcome	A	B	C	D	F
Probability	$\frac{30}{98}$	$\frac{46}{98}$	$\frac{10}{98}$	$\frac{3}{98}$	$\frac{9}{98}$

If a student from this group is selected at random, what is the probability that the student made

(a) an A? $P(\{A\}) = \frac{30}{98} = \frac{15}{49}$

(b) an A or a B? $P(\{A \cup B\}) = P(\{A, B\}) = P(\{A\}) + P(\{B\}) = \frac{30}{98} + \frac{46}{98} = \frac{76}{98}$

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EXAMPLE 7. *Let* $S = \{s_1, s_2, s_3, s_4, s_5\}$ *be the sample space associated with an experiment having the following probability distribution:*

not simple

Let $A = \{s_1, s_2, s_5\}, B = \{s_2, s_4, s_5\}, C = S$. *Compute:*

Outcome	s_1	s_2	s_3	s_4	s_5
Probability	$\frac{4}{15}$	$\frac{2}{15}$	$\frac{7}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

$= 1$

$1 - \frac{4}{15} - \frac{7}{15} - \frac{1}{15} - \frac{1}{15}$

(a) $P(C) = 1$

(b) $P(\emptyset) = 0$

(c) $P(\{s_2\}) = \frac{2}{15}$

(d) $P(A) = P(\{s_1\}) + P(\{s_2\}) + P(\{s_5\}) = = \frac{4}{15} + \frac{2}{15} + \frac{1}{15} = \frac{7}{15}$

(e) $P(B) = \frac{2}{15} + \frac{1}{15} + \frac{1}{15} = \frac{4}{15}$

(f) $P(B^c) = P(\{s_1, s_3\}) = \frac{4}{15} + \frac{7}{15} = \frac{11}{15}$

or $= 1 - P(B) = 1 - \frac{4}{15}$ //

(g) $P(A \cap B) = P(\{s_2\}) + P(\{s_5\}) = \frac{2}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5}$

$A \cap B = \{s_2, s_5\}$

(h) $P(A \cup B) =$ $\frac{4}{15} + \frac{2}{15} + \frac{1}{15} + \frac{1}{15} = \boxed{\frac{8}{15}}$

OR $1 - P(\{s_3\}) = 1 - \frac{7}{15} = \boxed{\frac{8}{15}}$

$A \cup B = \{s_1, s_2, s_4, s_5\} = \{s_3\}^c$

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EXAMPLE 8. *A pair of fair dice are rolled one time. Sample space for this experiment is below.*

	1	2	3	4	5	6
$S=\{$ 1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6) $\}$

F

E

$n(S) = 36$

(a) *Calculate the probability that the two dice show the same number.* = E

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \boxed{\frac{1}{6}}$$

(b) *Calculate the probability that the sum of the two die is 6.* = F

$$P(F) = \frac{n(F)}{n(S)} = \boxed{\frac{5}{36}}$$

(c) *Give the probability distribution for the sum of the die.* outcome

Outcome	2	3	4	5	6	7	8	9	10	11	12
Frequency Total # = 36	1	2	3	4	5	6	5	4	3	2	1
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(d) *Calculate the probability that the sum of the two die is a multiple of 5.*

$$P(\{5,10\}) = P(\{5\}) + P(\{10\}) = \frac{4}{36} + \frac{3}{36} = \boxed{\frac{7}{36}}$$

$n(S)=52$

EXAMPLE 9. *If a card is drawn at random from a standard 52-card deck, what is the probability that the card drawn is*

(a) a diamond

$$P(\{\Diamond\}) = \frac{n(\{\Diamond\})}{n(S)} = \frac{13}{52} = \boxed{\frac{1}{4}}$$

(b) a red card $P(\{R\}) = \frac{26}{52} = \boxed{\frac{1}{2}}$

(c) a king $P(\{K\}) = \frac{n(\{K\})}{52} = \frac{4}{52} = \boxed{\frac{1}{13}}$

EXAMPLE 10. *A ball is selected at random from an urn containing 3 yellow, 4 red, and 5 green balls. What is the probability that:*

Outcome	Y	R	G	
Frequency	3	4	5	= 12
Probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	

(a) *the ball is green?*

$P(\{G\}) = \frac{5}{12}$

(b) *the ball is green or red?* $P(G \cup R) = P(\{G, R\}) = P(G) + P(R) =$

$= \frac{4}{12} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4}$

(c) *the ball is purple?* $P(\{purple\}) = P(\varnothing) = 0$

(c) *the ball is yellow, green or red?* $P(Y \cup G \cup R) = P(S) = 1$