7.3: Rules of Probability

Recall:

$E \cap F = \emptyset$ means E and F are mutually exclusive.

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$ if E and F are NOT mutually exclusive.

Since

$$P(E) = \frac{n(E)}{n(S)}$$

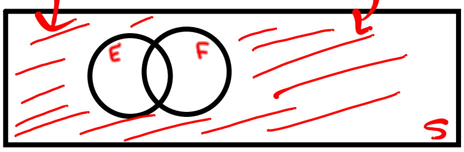
we have

$\boxed{P(E \cup F) = P(E) + P(F) - P(E \cap F)}$ if E and F are NOT mutually exclusive.

If E and F are mutually exclusive, then $P(E \cup F) = P(E) + P(F)$.

If E is an event of an experiment then $P(E^c) = 1 - P(E)$.

Remark: $P(E^c \cap F^c) = P((E \cup F)^c)$



EXAMPLE 1. *A single card is drawn from a standard deck of cards. Find these probabilities.*

(a) *P(Heart or a Queen)=* $P(H \cup Q) = P(H) + P(Q) - P(\underset{\heartsuit}{H} \cap \underset{Q}{Q}) =$

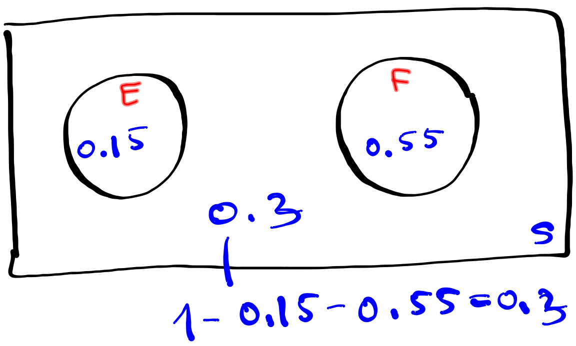
$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

(b) *P(not black Ace)=* $P(\{\text{Black Ace}\}^c) = 1 - P(\{\text{Black Ace}\}) =$

$$= 1 - \frac{n(\{\text{Black Ace}\})}{52} =$$

$$= 1 - \frac{2}{52} = \frac{50}{52} = \boxed{\frac{25}{26}}$$

EXAMPLE 2. *Let E and F be mutually exclusive events and suppose $P(E) = 0.15$ and $P(F) = 0.55$. Find:* $P(E \cap F) = \varnothing$



(a) $P(E \cap F) = \cancel{\varnothing}$

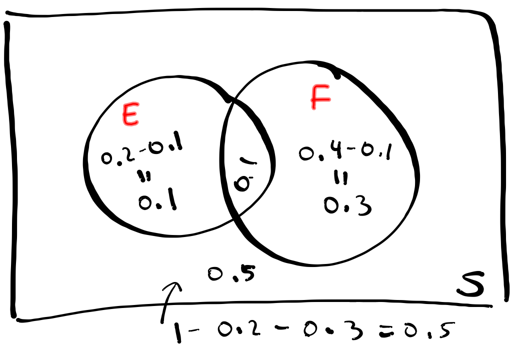
(b) $P(E \cup F) = 0.15 + 0.55 = 0.7$

(c) $P(E^c) = 0.3 + 0.55 = 0.85$

$\quad \parallel$ OR $1 - 0.15$ ✓

(d) $P(E^c \cap F^c) = P\left((E \cup F)^c\right) = 1 - P(E \cup F) = 1 - 0.7 = 0.3$

EXAMPLE 3. *Let E and F be two events of an experiment with sample space S. Suppose $P(E) = 0.2$, $P(F) = 0.4$, and $P(E \cap F) = 0.1$. Compute:*



(a) $P(E \cup F) = 0.5$

(b) $P(F^c) = 0.1 + 0.5 = 0.6$

(c) $P(F^c \cap E) = 0.1$

(d) $P(E^c \cap F^c) = P\left((E \cup F)^c\right) = 1 - P(E \cup F) = 1 - 0.5 = \boxed{0.5}$

EXAMPLE 4. *Roll an eight sided die. Suppose that any even number is twice as likely to happen as any odd number. Find the probability distribution.*

$P(\{1\}) = x \Rightarrow P(\{3\}) = P(\{5\}) = P(\{7\}) = x$

Outcome	1	2	3	4	5	6	7	8
Probability	x $\frac{1}{12}$	$2x$ $\frac{2}{12}$	x $\frac{1}{12}$	$2x$ $\frac{2}{12}$	x $\frac{1}{12}$	$2x$ $\frac{2}{12}$	x $\frac{1}{12}$	$2x$ $=1$ $\frac{2}{12}$

$\underbrace{x+2x} + \underbrace{x+2x} + \underbrace{x+2x} + \underbrace{x+2x} = 1$

$4 \cdot 3x = 1 \Rightarrow x = \frac{1}{12}$

$P(\{even\}) = \frac{2}{12} = \frac{1}{6}$ $\qquad$ $P(\{odd\}) = \frac{1}{12}$

EXAMPLE 5. *The quality control department of Star Communications, a manufacturer of video-game DVDs, has determined from records that 1.5% of the DVDs sold have video defects, 0.8% have audio defects, and 0.4% have both audio and video defects. What is the probability that a DVD purchased by a customer*

$$P(V) = \frac{1.5}{100} = 0.015$$

$$P(A) = \frac{0.8}{100} = 0.008$$

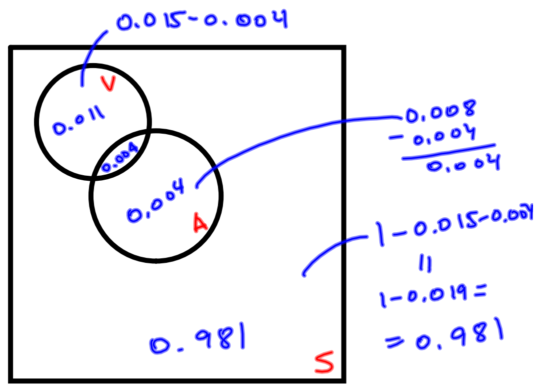
$$P(V \cap A) = \frac{0.4}{100} = 0.004$$

(a) *will have a video or audio defect?*

$$P(V \cup A) = 0.015 + 0.004 = \boxed{0.019}$$

(b) *will not have a video or audio defect?*

$$P((V \cup A)^c) = \boxed{0.981}$$



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EXAMPLE 6. *A company selected* 1000 *households at random and surveyed them to determine a relationship between income level and the number of television sets in a home. The information gathered is listed in the table:*

Annual Houshold Income, $	*0 TVs*	*1 TV*	*2 TVs*	*3 TVs*	*More than 3 TVs*	*TOTAL*
Less than 15,000	*0*	*38*	*53*	*11*	*0*	*102*
15,000-29,999	*0*	*72*	*78*	*15*	*1*	*166*
30,000-49,999	*2*	*112*	*130*	*80*	*12*	*336*
50,000-74,999	*13*	*87*	*80*	*60*	*21*	*261*
75,000 and higher	*27*	*35*	*28*	*25*	*20*	*135*
TOTAL	*42*	*344*	*369*	*191*	*54*	*1000*

TOTAL #

(a) *If a respondent is selected at random, find the probability that he or she owns* 0 *television sets.*

$$P(\{0\text{ TVs}\}) = \frac{n(\{0\text{ TVs}\})}{\text{total }\#} = \frac{42}{1000} = \boxed{0.042}$$

(b) *If a respondent is selected at random, find the probability that he or she owns a television set.*

at least 1 TVs

$$P(\text{not } 0\text{ TVs}) = P(\{0\text{ TVs}\}^c) = 1 - P(\{0\text{ TVs}\}) = 1 - \frac{42}{1000} = \boxed{0.958}$$

(c) *If a respondent is selected at random, find the probability that he or she is earning* \$30,000 – 49,999 *per year and owning one television set.*

$$\frac{112}{1000} = 0.112$$

(d) *If a respondent is selected at random, find the probability that he or she is earning* \$15,000 – 29,999 *per year and owning at most 2 television sets.*

$$\frac{0 + 72 + 78}{1000} = 0.15$$

(e) *If a respondent is selected at random, find the probability that he or she owns at least* 2 *television sets.*

$$\frac{369 + 191 + 54}{1000} = \boxed{0.614}$$

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