7.4: Use of Counting Techniques in Probability

RECALL: The **probability** of an event E happening is the number of ways that specific event can happen divided by the number of possible outcomes in the whole experiment, i.e.

$$\boxed{P(E) = \frac{n(E)}{n(S)} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}.}$$

experiment

EXAMPLE 1. *A fair coin is tossed seven times. Find the probability that the coin lands on tails*

$\overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2} = 2^7 = n(S)$

(a) *at most 4 times.*

$E = \{T \text{ at most } 4 \text{ times}\}$

4T for 7 places and 3H for 3 places
or 3T for 7 places and 4H for 4 places
or 2T — " — — " — 5H for 5 — " —
or 1T — " — 6H for 6 — " —
or 0T — " — 7H for 7 places

$C(7,4) \cdot C(3,3)$
$+ C(7,3) \cdot C(4,4)$
$+ C(7,2) \cdot C(5,5)$
$+ C(7,1) \cdot C(6,6)$
$+ C(7,0) \cdot C(7,7)$

$99 = n(E)$

$P(E) = \frac{n(E)}{n(S)} = \frac{99}{2^7} \approx 0.77$

(b) *on the first toss and the last toss.* $\{ \quad \} = F$

$\overline{1}^{T} \cdot \underbrace{\overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2} \cdot \overline{2}}_{T \text{ or } H} \cdot \overline{1}^{T} = 2^5 = n(F)$

$P(F) = \frac{n(F)}{n(S)} = \frac{2^5}{2^7} = \frac{1}{2^2} = \frac{1}{4} = 0.25$

EXAMPLE 2. *A type of license plate has three letters followed by three digits. If one of these plates is select at random, find the probability that* $\{ \quad \} = E$

$\overset{L}{\overline{26}} \cdot \overset{L}{\overline{26}} \cdot \overset{L}{\overline{26}} \cdot \overset{D}{\overline{10}} \cdot \overset{D}{\overline{10}} \cdot \overset{D}{\overline{10}} = 26^3 \cdot 10^3 = n(S)$

(a) *The license plate has all vowels.*

$\overset{V}{\overline{5}} \cdot \overset{V}{\overline{5}} \cdot \overset{V}{\overline{5}} \cdot \overset{D}{\overline{10}} \cdot \overset{D}{\overline{10}} \cdot \overset{D}{\overline{10}} = 5^3 \cdot 10^3$

$P(E) = \frac{n(E)}{n(S)} = \frac{5^3 \cdot \cancel{10^3}}{26^3 \cdot \cancel{10^3}} = \left(\frac{5}{26}\right)^3 \approx 0.007$

$\{ \quad \} = F$

(b) *The letters are selected from the set A,B,C,D,E,F,G, the digit zero is not used and no letter is repeated.*

$\overset{L}{\overline{7}} \cdot \overset{L}{\overline{6}} \cdot \overset{L}{\overline{5}} \cdot \overset{D}{\overline{9}} \cdot \overset{D}{\overline{9}} \cdot \overset{D}{\overline{9}} = n(F)$

$P(F) = \frac{n(F)}{n(S)} = \frac{7 \cdot 6 \cdot 5 \cdot 9^3}{26^3 \cdot 10^3} \approx 0.0087$

Jun 17-4:45 PM

EXAMPLE 3. *Three cards are drawn at random from a deck of 52 cards* (experiment). *What is the probability that:*

$C(52,3) = n(S)$

(a) {*all three cards are jacks*} $= E$ (event)

select 3 J from total 4 J $\Rightarrow n(E) = C(4,3) = 4$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{C(52,3)} \approx 1.8 \cdot 10^{-4}$$

(b) {*at least two of the cards are jacks*} $= F$

2 J from 4 J and 1 J^c from $52-4=48$

OR 3 J from 4 J

$C(4,2)\cdot C(48,1) + C(4,3)$

$n(F) = 292$

$$P(F) = \frac{n(F)}{n(S)} = \frac{292}{C(52,3)} = \frac{73}{5525} \approx \boxed{0.013}$$

EXAMPLE 4. *A box contains 8 red, 7 white, and 6 yellow chalks. What is the probability that in a sample of 7*

Experiment: draw 7 chalks from $8+7+6=21$

$n(S) = C(21,7) = 116{,}280$

(a) {*all the chalks are the same color*} $= E$ (event)

Draw 7 R from 8 R

or draw 7 W from 7 W

$\Rightarrow n(E) = C(8,7) + C(7,7) = 8+1 = 9$

$$P(E) = \frac{9}{116280} \approx 7.7 \cdot 10^{-5}$$

(b) {*exactly 3 red and at least 3 white chalks were chosen*} $= F$

3 R from 8 R and 3 W from 7 W and 1 Y from 6 Y

OR 3 R from 8 R and 4 W from 7 W

$$n(F) = C(8,3)\,C(7,3)\,C(6,1) + C(8,3)\cdot C(7,4)$$

$$P(F) = \frac{n(F)}{116\,280} \approx 0.118$$

Jun 17-4:48 PM

EXAMPLE 5. *Five cards are selected at random without replacement from a well-shuffled deck of 52 playing cards. Find the probability of the cards being in the same suit.*

Experiment: select 5 from 52 $\quad n(S) = C(52,5)$

Event : $E = \{$cards in the same suit$\}$

select 1 suit from 4 : $C(4,1) = 4$
and
select 5 from 13 : $C(13,5)$

$\Rightarrow n(E) = 4C(13,5)$

$$P(E) = \frac{4C(13,5)}{C(52,5)} \approx 0.002$$

EXAMPLE 6. (*Twenty cards are selected at random without replacement from a well-shuffled deck of 52 playing cards*) *Find the probability of* $\underbrace{\textit{the cards being in the same suit.}}_{\text{impossible event}}$ $= E = \varnothing$

$n(S) = C(52,20)$

$n(\varnothing) = 0$

$$P(E) = \frac{n(E)}{n(S)} = \frac{0}{C(52,20)} = 0$$

Jun 17-4:49 PM

experiment $n(S) = C(21,12)$

EXAMPLE 7. *An exam consist of 21 questions in which 12 of them must be answered. What is the probability that a student answered at least 4 of the first 7 questions and exactly 3 of the last 7 questions?* = E

	1-7	and 8-14	and 15-21	
	4	5	3	=12
or	5	4	3	=12
or	6	3	3	=12
or	7	2	3	=12

$$n(E) = C(7,4)C(7,5)C(7,3) + C(7,5)C(7,4)C(7,3) + C(7,6)C(7,3)C(7,3) + C(7,7)C(7,2)C(7,3) = 60,760$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{60,760}{C(21,12)} \approx 0.2067$$

EXAMPLE 8. (a) *What is the probability that* [redacted] all *of 9 people were born in a different days of the year? (Assume that none of the 9 people was born on February 29 of a leap year and that each of them is equally likely to have any of the 365 days of a year as his or her birthday.)*

$$\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365}\cdot\overline{365} = 365^9 = n(S)$$

$E = \{$ noone borns at the same day $\}$

$$\overline{365}\cdot\overline{364}\ \overline{\ \ }\cdot\overline{\ \ }\cdot\overline{\ \ }\cdot\overline{\ \ }\cdot\overline{\ \ }\cdot\overline{\ \ }\cdot\overline{357} = n(E)$$

$$P(E) = \frac{\cancel{365}\cdot 364\cdot 363\cdot 362\cdot 361\cdot 360\cdot 359\cdot 358\cdot 357}{365^{\cancel{9}\,8}} \approx 0.905$$

(b) *What is the probability that* $\{$*at least two of 9 people have the same birthday*$\} = F$?

$F = E^c$

$P(F) = P(E^c) = 1 - P(E) \approx 1 - 0.905 \approx 0.095$

EXAMPLE 9. *A shelf in Office Max contains 80 cartridges for HP printer. Six of the cartridges are defective. If a customer select 2 cartridges at random from the shelf, what is the probability that both are defective?*

Experiment: select 2 of 80 | $n(S) = C(80, 2)$

Event $E = \{$ both are defective $\}$ | $n(E) = C(6, 2)$

select 2 of 6

$$P(E) = \frac{n(E)}{n(S)} = \frac{C(6,2)}{C(80,2)} = \frac{15}{3160} \approx 0.0047$$