7.5: Conditional Probability and Independent Events

EXAMPLE 1. *Two cards are drawn from a deck of 52 without replacement.*

(a) *What is the probability of that the first card drawn is a King?*

(b) *What is the probability that the second card drawn is a King giving that the first card drawn was a King?*

(c) *What is the probability the second card drawn is a King giving that the first card drawn was not a King?*

DEFINITION 2. *If A and B are events in an experiment and $P(A) \neq 0$, then the* ***conditional probability*** *that the event B will occur, given that the event A has already occurred, is*

$$P(B|A) = \frac{P(A \cap B)}{P(A)}.$$

EXAMPLE 3. *The data below were obtained from the financial aid office of a certain university:*

	Receiving Fin. Aid	*Not Receiving Fin. Aid*	*TOTAL*
Undergraduates	*4,222*	*3,898*	*8,120*
Graduates	*1,879*	*731*	*2,610*
TOTAL	*6,101*	*4,629*	*10,730*

Let A be the event that a student selected at random from this university is an undergraduate student, and let B be the event that a student selected at random is receiving financial aid. Find each of the following probabilities:

1. $P(A) =$
2. $P(B) =$
3. $P(A \cap B) =$
4. $P(B|A) =$
5. $P(B|A^c) =$

EXAMPLE 4. *Given the probability distribution*

S	a	b	c	d	e	f
p	0.1	0.2	0.3	0.15	0.08	0.17

and the events $A = \{a, c, d, e\}$, $B = \{b, d, f\}$, $C = \{a, b, d\}$.
Find each of the following probabilities:

1. $P(A|C) =$

2. $P(C|B) =$

EXAMPLE 5. *During lunch period at a certain middle school* 25% *of the students buy vanilla ice-cream,* 50% *buy chocolate ice-cream, and* 5% *buy both products. Find the probability that*

(a) *a student selected at random will buy vanilla ice-cream if he already bought chocolate ice cream.*

(b) *a student selected at random will not buy chocolate ice-cream if he already bought vanilla ice cream.*

EXAMPLE 6. *A red die and a blue die are rolled at the same time (they are both fair die). What is the probability the sum of the numbers falling uppermost is* 6*, if it is known that the number falling uppermost on the red die is* 4.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

• INDEPENDENT EVENTS

Two events A and B are independent if the outcome of one does not affect the outcome of the other.

DEFINITION 7. *If A and B are* ***independent events*** *then*

$$P(A|B) = P(A), \text{ and } P(B|A) = P(B).$$

TEST for the Independence of Two Events: *Two events, A and B are independent iff*

$$P(A \cap B) = P(A) \cdot P(B)$$

EXAMPLE 8. *Are the events A and B from Example 3 independent events?*

REMARK 9. Do not confuse *independent* events with *mutually exclusive* events.

EXAMPLE 10. *A group of* 30 *high school students was asked about the types of pets they have, specifically, whether they have a cat or dog. The survey revealed the following information:*

4 *students have only cat;* 12 *students have only dog;* 6 *students have cat and dog.*

Determine whether the event that the student has cat and the event that the students has dog are independent?

EXAMPLE 11. *Let* E, F, G *are three independent events of an experiment with* $P(E) = 0.4$, $P(F) = 0.75$ *and* $P(G) = 0.3$. *Compute each of the following:*

(a) $P(E^c \cap F)$

(b) $P(E^c \cup F)$

(c) $P(F|G)$

EXAMPLE 12. *A personnel manager at a certain company claims that he approves qualified applicants for a certain job* 80% *of the time; he rejects an unqualified person* 75% *of the time. It is known that* 70% *of applicants for this job are qualified.*

(a) *Construct the* probability tree *that represents this experiment.*

(b) *What is the probability that an applicant is not approved?*

(c) *What is the probability that an applicant is approved for the job if he/she is unqualified?*

(d) *What is the probability that an applicant is unqualified if he/she is approved for the job?*

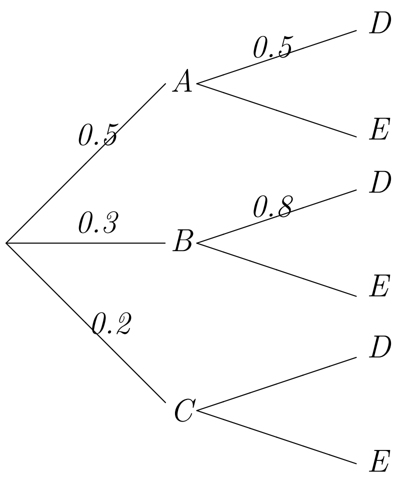
EXAMPLE 13. *Laura has a box (B) and a jar (J). Box contains* 4 *red,* 3 *green and* 2 *black marbles. Jar contains* 8 *red and* 5 *green marbles. The experiment is to draw a single marble. It is known that Laura is twice as likely to draw a marble from box as from jar.*

(a) *Construct the* probability tree *that represents this experiment.*

(b) *Compute*

- $P(r|B)$
- $P(g|J)$
- $P(b|J)$
- $P(J \cap r)$
- $P(B \cap g)$
- $P(g)$

EXAMPLE 14. **(a)** *Complete the tree diagram below if* $P(C \cap D) = 0.08$.



(b) *Compute:*

- $P(E|A) =$
- $P(A \cup D) =$

- $P(B|D) =$