7.6: Bayes Theorem

In Section 7.5 weve been calculating a priori probabilities - the likelihood an event will occur. In this section we are going to look at a *posteriori probabilities* - calculating probability after an outcome has been observed.

Bayes' Theorem: $P(B|A) = \frac{P(A \cap B)}{P(A)}$.

EXAMPLE 1. Using the tree diagram below, find:



- (a) $P(D^c)$
- **(b)** *P*(*D*)

(c) P(D|C)

(d) P(C|D)

EXAMPLE 2. Box A has 5 purple, 3 yellow, and 7 red items in it. Box B has 4 yellow and 6 red items in it. An item is drawn from Box A and transferred to Box B. An item is then drawn from Box B. The color of item drawn from each box is recorded.

(a) Draw a tree diagram representing this experiment.

- (b) What is the probability that both items are purple?
- (c) What is the probability that the item drawn from the box A was yellow or the item drawn from the Box B was red?
- (d) What is the probability that the item drawn from the box B is purple if the item drawn from the Box A was yellow?
- (e) What is the probability that the transferred item was red if a yellow item was selected from the Box B?

EXAMPLE 3. Two cards are drawn from a deck of 52. What is the probability the 1st card is a face card, given that the 2nd card is an ace?

EXAMPLE 4. If a certain disease is present, then a blood test will reveal it 95% of the time. But the test will also indicate the presence of the disease 2% of the time when in fact the person tested is free of that disease; that is, the test gives a false positive 2% of the time. If 0.3% of the general population actually has the disease, what is the probability that a person chosen at random from the population

(a) has the disease given that he/she tested positive?

(b) does not have the disease given that they took the test twice and got positive results both times.

EXAMPLE 5.

Age Group	% of Insured Drivers	Accident rate, %
Under 25	16	5.5
25-44	40	2.5
45-64	30	2
65+	14	4

What is the probability that an insured driver selected at random

(a) will be involved in an accident?

(b) who is involved in an accident is under 25?