## 7.6: Bayes Theorem

In Section 7.5 weve been calculating a priori probabilities - the likelihood an event will occur. In this section we are going to look at a posteriori probabilities - calculating probability after an outcome has been observed.

Bayes' Theorem: $P(B \mid A)=\frac{P(A \cap B)}{P(A)}$.
EXAMPLE 1. Using the tree diagram below, find:

(a) $P\left(D^{c}\right)$
(b) $P(D)$
(c) $P(D \mid C)$
(d) $P(C \mid D)$

EXAMPLE 2. Box $A$ has 5 purple, 3 yellow, and 7 red items in it. Box B has 4 yellow and 6 red items in it. An item is drawn from Box $A$ and transferred to Box B. An item is then drawn from Box B. The color of item drawn from each box is recorded.
(a) Draw a tree diagram representing this experiment.
(b) What is the probability that both items are purple?
(c) What is the probability that the item drawn from the box $A$ was yellow or the item drawn from the Box B was red?
(d) What is the probability that the item drawn from the box $B$ is purple if the item drawn from the Box A was yellow?
(e) What is the probability that the transferred item was red if a yellow item was selected from the Box B?

EXAMPLE 3. Two cards are drawn from a deck of 52. What is the probability the 1 st card is a face card, given that the 2nd card is an ace?

EXAMPLE 4. If a certain disease is present, then a blood test will reveal it $95 \%$ of the time. But the test will also indicate the presence of the disease $2 \%$ of the time when in fact the person tested is free of that disease; that is, the test gives a false positive $2 \%$ of the time. If $0.3 \%$ of the general population actually has the disease, what is the probability that a person chosen at random from the population
(a) has the disease given that he/she tested positive?
(b) does not have the disease given that they took the test twice and got positive results both times.

EXAMPLE 5.

| Age Group | \% of Insured Drivers | Accident rate, \% |
| :--- | :---: | :---: |
| Under 25 | 16 | 5.5 |
| $25-44$ | 40 | 2.5 |
| $45-64$ | 30 | 2 |
| $65+$ | 14 | 4 |

What is the probability that an insured driver selected at random
(a) will be involved in an accident?
(b) who is involved in an accident is under 25?

