8.1: Distributions of Random Variables

A random variable is a rule that assigns a number to each outcome of a sample space.

EXAMPLE 1. *Let X be the number of girls in a three-child family.*

$S = \{bbb, bbg, bgb, gbb, ggb, gbg, bgg, ggg\}$

(a) *What are the values of the random variable X?*

$X = \{0, 1, 2, 3\}$ domain of X

(c) *Give the probability distribution for X.*

$x \in X$ value from the domain of X

x	0	1	2	3	$=8$
frequency	1	3	3	1	
$P(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$	

TYPES OF RANDOM VARIABLES:

8.2 - 8.4

- **Finite Discrete** Random Variable that assumes only a finite number of values. (You can write ALL possible values of the random variable that stops.)

$X = \{0, 2, 4, 8, 15\}$ domain → finite set

- **Infinite Discrete** Random Variable takes on infinitely many values, which may be arranged in a sequence. (You can write all possible values of a random variable in a list of numbers that has a pattern and goes on forever.)

$X = \{1, 3, 5, 6, 7, \dots\}$ domain → sequence; all possible values

8.5 - 8.6

- **Continuous Random Variable** may assume an interval of real numbers.

There is nothing like an exact observation in the continuous variable.

$X = \{x \mid 3 \le x \le 14.7\}$ domain → interval

EXAMPLE 2. *Classify these random variables. Give the values of the random variable (domain).*

(a) *Three cards are drawn from a standard deck of 52. Let X be the random variable denoting the number of diamonds that are drawn. What is the domain of X?*

$X = \{0, 1, 2, 3\}$ domain — finite discrete

(b) *A bag contains 3 red, 6 blue, and 4 white marbles. Marbles are drawn one at a time without replacement until a red one is drawn. Let X be the random variable denoting the number of marbles drawn in one trial of this experiment.*

$X = \{1, 2, \dots, 11\}$ domain — finite discrete

(c) *Let X be the number of times you roll a dice until a 4 appears.*

$X = \{1, 2, 3, \dots\}$ infinite discrete

(d) *Let X denote the number of minutes a person waits (in one particular day) in line to pull football tickets.*

$X = \{x \mid 0 \le x \le 24 \cdot 60\}$ domain — continuous random variable.

EXAMPLE 3. *Two cards are drawn from a well-shuffled deck of 52 playing cards. Let X denote the number of aces drawn. Find $P(X = 2)$.*

$X = \{0, 1, 2\}$

event E = {# of aces drawn}

$n(E) = C(4,2)$

$n(S) = C(52,2)$

$P(X=2) = P(E) = \frac{C(4,2)}{C(52,2)} = \frac{6}{1326} \approx 0.005$

using random variables. — using events.

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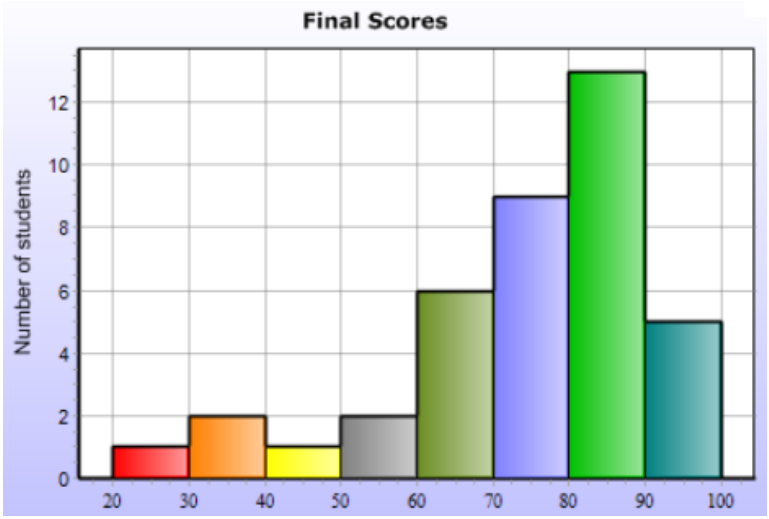
DEFINITION 4. *A* ***histogram*** *is a way to present the probability distribution of a discrete random variable.*

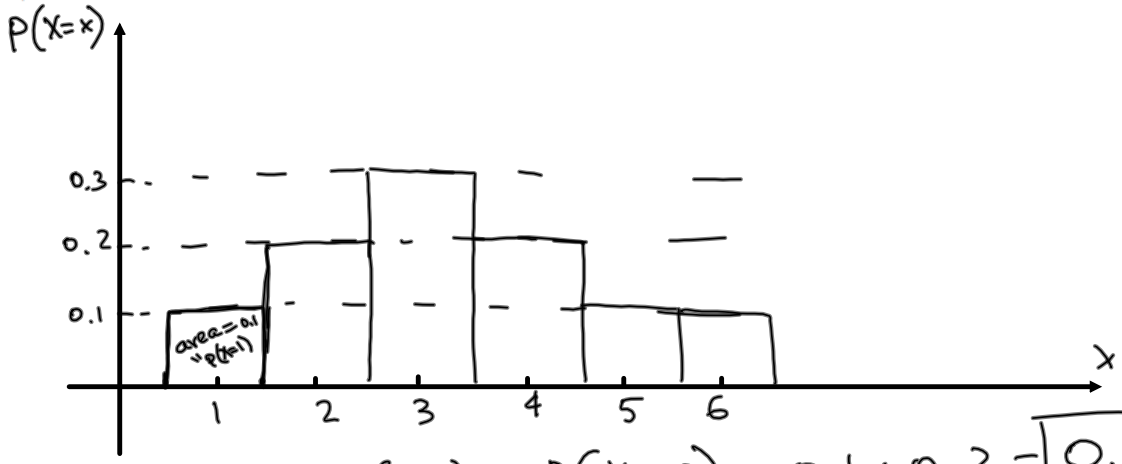


http://www.cyber-wit.com/images/gallery/histogram1_b.png

EXAMPLE 5. *The probability distribution of the random variable X is shown:*

x	1	2	3	4	5	6
$P(X=x)$	0.1	0.2	0.3	0.2	0.1	0.1

(a) *Draw the histogram for the random variable X.*

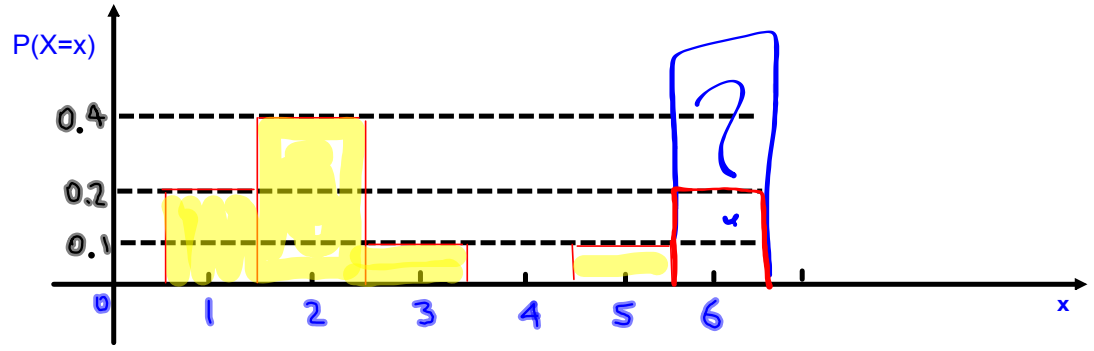


(b) *Compute* $P(X<3)$ $= P(X=1) + P(X=2) = 0.1 + 0.2 = \boxed{0.3}$

(c) $P(X \le 4)$ $= 0.1 + 0.2 + 0.3 + 0.2 = \boxed{0.8}$
$1, 2, 3, 4$

(d) $P(1<X\le 6)$ $= 1 - P(X=1) = 1 - 0.1 = \boxed{0.9}$

EXAMPLE 6. *The following histogram (your teacher just drew :)) is only missing the rectangle at $x = 6$.*



(a) *Find* $P(X = 6)$ = 1 − 0.2 − 0.4 − 0.1 − 0 − 0.1 = 0.2

(b) *Give the probability distribution for* X.

x	1	2	3	4	5	6
$P(X=x)$	0.2	0.4	0.1	0	0.1	0.2

= 1

(c) *Find* $P(2 \leq X < 5)$ = 0.4 + 0.1 + 0 = 0.5

EXAMPLE 7. *The rates paid by thirty financial institutions on a certain day for money-market deposit accounts are shown in the accompanying table:*

Rate, %	*6*	*6.25*	*6.55*	*6.56*	*6.58*	*6.60*	*6.65*	*6.85*
Institutions	*1*	*7*	*7*	*1*	*1*	*8*	*3*	*2*
$P(X=x)$	$\frac{1}{30}$	$\frac{7}{30}$	$\frac{7}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{8}{30}$	$\frac{3}{30}$	$\frac{2}{30}$

$= 30$

Let the random variable X denote the interest paid by a randomly chosen financial institution on its money-market deposit accounts. Find the probability distribution associated with these data.

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