8.2: Expected Value

- MEAN

    The **average**, or **mean**, of a set of n numbers

$$x_1, x_2, x_3 \ldots, x_n$$

is denoted by $\overline{x}$, where

$$\overline{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

EXAMPLE 1. *In a class of* 10, *the grades on the last test were:*

    44   53   73   76   76   76   80   85   95   95

    What was the class average for this test?

DEFINITION 2. *Let X denote the random variable with values of $x_1, x_2, x_3 \ldots, x_n$ and the probabilities of $p_1, p_2, p_3, \ldots, p_n$ respectively. Then the **expected value** of X is:*

$$E(X) = x_1 p_1 + x_2 p_2 + x_3 p_3 + \ldots + x_n p_n.$$

REMARK 3. A game is said to be *fair* when $E(X) = 0$.

EXAMPLE 4. *The chef at the local Italian restaurant recorded the weekly consumption of pasta over* 16 *week period.*

Qts Pasta	100	120	140	160	180
Weeks	2	3	4	4	3

(a) *Find the average weekly consumption of pasta at this restaurant.*

(b) *Let X denote the number of quarts of salsa in a week at the restaurant. Compute the expected value of X.*

EXAMPLE 5. *A sample of 7 DVD's is selected from a lot of 25 DVD's, of which 2 are defective. What is the expected number of defective DVD's.*

EXAMPLE 6. *The Island Club is holding a fund-raising raffle. Ten thousand tickets have been sold for $2 each. There will be a first prize of $3000, 3 second prizes of $1000 each, 5 third prizes of $500 each, and 20 consolation prizes of $100 each. Letting X denote the net winnings (that is, winnings less the cost of the ticket) associated with the tickets, find $E(X)$. Interpret your results.*

EXAMPLE 7. *Bill, Laura and Amy play a card game with a standard deck of 52 cards. Bill selects a card from a well-shuffled deck and receives $A from Laura and $A from Amy if the card selected is a diamond; otherwise Bill pays Laura and Amy 3 dollars each. Determine the value of A if the game is to be fair.*

EXAMPLE 8. *A man purchased a $10,000 1-yr term life insurance policy for $150. Assuming that the probability that he will live another year is 0.993, find the company's expected gain.*

EXAMPLE 9. *Bob wishes to purchase a 5-yr term life-insurance policy that will pay the beneficiary $25,000 in the event he dies in the next 5 years. The probability Bob lives the next 5 years is 0.99. What is the minimum amount Bob can expect to pay for this policy? (NOTE: the minimum premium occurs when the insurance company's expected profit is zero.)*

• ODDS

The odds that an event E will occur, odds in favor of E, are given as a to b or $a : b$ where a and b are integers and the fraction $\dfrac{a}{b}$ is in reduced form.

Computing ODDS from probability

If $P(E)$ is the probability of an event, E, occurring, then

• the **odds in favor** of E occurring are

$$\frac{P(E)}{1 - P(E)} = \frac{P(E)}{P(E^c)}, \quad P(E) \neq 1.$$

• the **odds against** E occurring are

$$\frac{1 - P(E)}{P(E)} = \frac{P(E^c)}{P(E)}, \quad P(E) \neq 0.$$

EXAMPLE 10. *Find the odds in favor of drawing a face card from a deck of 52.*

EXAMPLE 11. *If the probability David will win his tennis match tomorrow is* 0.75. *What are the odds against his winning his match tomorrow?*

If the odds in favor of an event, E, occurring are a to b, then the probability of E occurring is
$$P(E) = \frac{a}{a + b}.$$

EXAMPLE 12. *The odds in favor of the event E are 5 to 11.*

(a) *Find the odds against E.*

(b) *Find $P(E)$ and $P(E^c)$.*

EXAMPLE 13. *Roger feels that the odds in favor of a business deal going through are 9 to 5. What is the (subjective) probability that this deal will* not *materialize?*

• MEDIAN and MODE

The **median** of a data set is the middle value of a set of data that is arranged in increasing or decreasing order. If there are 2 such values, the median is the mean of those 2 numbers.

The **mode** of a data set are the values that have the largest frequencies (occur most often).

EXAMPLE 14. *For the data set:*
    50   46   43   42   45   49   48   40   52

(a) *What is the median?*

(b) *What is the mode?*

EXAMPLE 15. *For the data set:*
    2   3   4   4   4   5   5   5   11   11

(a) *What is the median?*

(b) *What is the mode?*

EXAMPLE 16. *The frequency distribution of the hourly wage rates (in dollars) among blue-collar workers in a certain factory is given in the following table. Find the mean (or average) wage rate, the mode, and the median wage rate of these workers.*

Wage Rate	10.70	10.80	10.90	11.00	11.10	11.20
Frequency	60	90	75	120	60	45