

8.3: Variance and Standard Deviation

EXAMPLE 1. Find the mean and median of these data sets.

(a) 50, 50, 50, 110, 110, 110

$$\bar{x} = \frac{3 \cdot 50 + 3 \cdot 110}{6} = 80$$

$$\text{median} = \frac{50 + 110}{2} = 80$$

(b) 70, 75, 75, 80, 85, 85, 90

$$\bar{x} = 80 \quad \text{median} = 80$$

Variance of the random variable X , denoted $\text{Var}(x)$, measures the dispersion (or spread) of the data.

$$\text{Var}(X) = \sigma^2$$

Standard Deviation, denoted σ , is another measure of dispersion (or spread) of a probability distribution about its mean:

$$\sigma = \sqrt{\text{Var}(X)}$$

Finding expected value and standard deviation using the calculator:



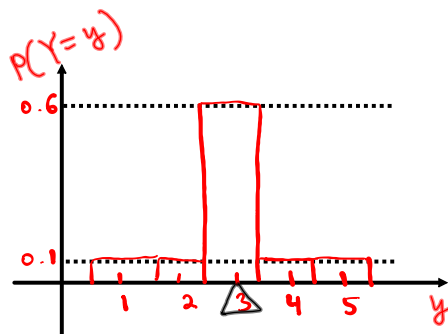
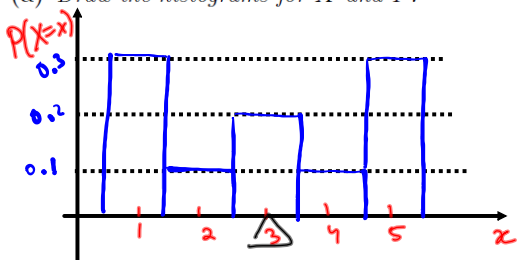
1. Enter the x -values into $L1$ and the corresponding probabilities into $L2$ (STAT→1:Edit).
2. On the homescreen type 1-Var Stats $L1, L2$ (STAT→CALC→1:1-Var Stats)
3. \bar{x} is the expected value
4. σx is the standard deviation
5. To find variance, you would need to recall that $\text{Var}(X) = \sigma^2$.

EXAMPLE 2. The random variables X and Y have the following probability distributions:

x	$P(X = x)$
1	0.3
2	0.1
3	0.2
4	0.1
5	0.3

y	$P(Y = y)$
1	0.1
2	0.1
3	0.6
4	0.1
5	0.1

(a) Draw the histograms for X and Y .



(b) Find the mean, variance and standard deviation of X .

$$\bar{x} = 3, \quad \sigma_x = 1.6 \Rightarrow \text{Var}(X) = 2.56$$

(c) Find the mean, variance and standard deviation of Y .

$$\text{mean } \bar{y} = 3, \quad \sigma_y = 1 \Rightarrow \text{Var}(Y) = 1^2 = 1$$

EXAMPLE 3. In a class of 100 students, define X as the random variable whose value represents the students grade in Math 141, $A = 4, B = 3, C = 2, D = 1, F = 0$, etc. Compute the mean, and standard deviation.

Scenario	A	B	C	D	F	
x	4	3	2	1	0	L1
freq. of occurrence	15	29	40	12	4	L2

$$\bar{x} \approx 2.39$$

$$\sigma \approx 1$$

EXAMPLE 4. Amy is considering investing \$5,000 in 2 mutual funds. The anticipated return from price appreciation and dividends (in hundred of dollars) are described by the following probability distributions:

MUTUAL FUND A

Returns	Probability
-2	0.25
4	0.45
5	0.3

L1 L2

MUTUAL FUND B

Returns	Probability
-2	0.2
3	0.4
4	0.4

L1 L2

(a) Find the mean and variance with the returns for each fund.

Fund A: $\mu_A = 2.8$
 $\sigma_A \approx 2.8 \Rightarrow \text{Var}(X_A) \approx 2.8^2 = 7.84$

Fund B: $\mu_B = 2.4$
 $\sigma_B = 2.24 \Rightarrow \text{Var}(X_B) = 2.24^2 = 5.02$

For A: mean $\$ 2.8 \cdot 100 = \280

For B: mean $\$ 2.4 \cdot 100 = \240

(b) Which investment would provide Amy with the higher expected return?

Fund A, because $\bar{x}_A > \bar{x}_B$

(c) In which investment would the element of risk be less?

For fund B because the probability distribution for B has the smallest variance

$$\text{Var}(X_A) > \text{Var}(X_B)$$