

## Appendix D: Trigonometry

**Angles.** Angles can be measured in degrees or radians.

**Everything in most calculus classes will be done in radians.**

The angle given by a complete revolution contains  $360^0$ , or  $2\pi$  radians, i.e.  $360^0 = 2\pi$  rad.

Hence,

$$\pi \text{ rad} = 180^0$$

and

$$1 \text{ rad} = \left(\frac{180}{\pi}\right)^0, \quad 1^0 = \frac{\pi}{180} \text{ rad.}$$

EXAMPLE 1. Convert  $-315^0$  to radians.

EXAMPLE 2. Convert  $\frac{5\pi}{6}$  to degrees.

For basic angles we have the following table (Know it!)

Degree	0	30	45	60	90	180	270	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$

**Six trigonometric functions** and how they relate to each other.

$$\begin{array}{ll} \cos x & \sin x \\ \tan x = \frac{\sin x}{\cos x} & \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x} \\ \sec x = \frac{1}{\cos x} & \csc x = \frac{1}{\sin x} \end{array}$$

For an acute angle  $\theta$  all the trig functions can be defined as ratios of sides of a right triangle:

$$\begin{array}{ll} \cos \theta = \frac{\text{adj}}{\text{hyp}} & \sin \theta = \frac{\text{opp}}{\text{hyp}} \\ \tan \theta = \frac{\text{opp}}{\text{adj}} & \cot \theta = \frac{\text{adj}}{\text{opp}} \\ \sec \theta = \frac{\text{hyp}}{\text{adj}} & \csc \theta = \frac{\text{hyp}}{\text{opp}} \end{array}$$

### Pythagorean Theorem

The exact trigonometric ratios for certain angles can be read from the following two special triangles.

$$\begin{array}{lll} \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} & \sin \frac{\pi}{6} = \frac{1}{2} & \sin \frac{\pi}{3} = \\ \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} & \cos \frac{\pi}{6} = & \cos \frac{\pi}{3} = \\ \tan \frac{\pi}{4} = & \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} & \tan \frac{\pi}{3} = \end{array}$$

## Using of unit circle

EXAMPLE 3. Evaluate each of the following:

(a)  $\sin \frac{5\pi}{6}$

(b)  $\sin \left( -\frac{5\pi}{6} \right)$

(c)  $\cos \left( \frac{4\pi}{3} \right)$

(d)  $\cos \left( -\frac{4\pi}{3} \right)$

(e)  $\tan \left( -\frac{\pi}{4} \right)$

EXAMPLE 4. If  $\sin \theta = \frac{2}{3}$  and  $0 < \theta < \pi/2$ , find the other five trigonometric functions of  $\theta$ .

Know these identities

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta \quad \cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta) \quad \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

EXAMPLE 5. Find all values of  $x$  in the interval  $[0, 2\pi]$  such that

(a)  $\sin 2x = \cos x$

(b)  $\sin x = \frac{3}{2}$

EXAMPLE 6. If  $\sec y = \frac{5}{4}$ , where  $0 < y < \pi/2$ , evaluate  $\sin 2y$ .

### Graphs of the trig functions

- $y = \sin x$

- $y = \cos x$

- $y = \tan x$

EXAMPLE 7. *Graph the following functions:*

(a)  $f(x) = \sin\left(x + \frac{\pi}{4}\right)$

(b)  $f(x) = 2 - \cos x$