Final Exam Practice

In addition to working this problem set, it is advised that you work the first three exams and corresponding sample tests, and quizzes/activities, as well as Lecture Notes.

1. Given $\mathbf{a} = \langle -2, 3\rangle, \mathbf{b} = \langle 6, 1\rangle, \mathbf{c} = 2\mathbf{i}$. Find:
   (a) $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$
   (b) a unit vector having the same direction as $\mathbf{a}$
   (c) Find the angle between $\mathbf{a}$ and $\mathbf{b}$
   (d) a unit vector that is orthogonal to $\mathbf{a} + \mathbf{b}$
   (e) scalars α and β such that $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b}$
2. Find a vector equation of the line containing the points $(-1, 1)$ and $(2, 5)$.
3. Find a unit vector perpendicular to the line described by the parametric equations $x = -4t + 1$, $y = 6t + 5$.
4. Find the work done by a force of $30N$ acting in the direction $N30^oW$ (i.e. 30^o west of the northerly direction) in moving an object $6m$ due west.
5. Determine whether the vectors $\langle 1, 2\rangle$ and $\langle -2, 3\rangle$ are orthogonal, parallel, or neither.
6. What is the limit:
   (a) $\lim\limits_{\theta\to\pi/3} \dfrac{\cos\theta - \frac{1}{2}}{\theta - \pi/3}$
   (b) $\lim\limits_{h\to 0} \dfrac{(2+h)^6 - 64}{h}$
   (c) $\lim\limits_{h\to 0} \dfrac{\sin(\pi/4 + h) - \sin(\pi/4)}{h}$
7. Compute the following limits:
   (a) $\lim\limits_{x\to -6^+} \dfrac{x}{x+6}$
   (b) $\lim\limits_{x\to 16} \dfrac{4 - \sqrt{x}}{x - 16}$
   (c) $\lim\limits_{x\to 8^-} \dfrac{|x-8|}{x-8}$
   (d) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^2-9}}{2x-6}$
   (e) $\lim\limits_{x\to\infty} (\sqrt{x^2+x+1} - \sqrt{x^2-x})$
   (f) $\lim\limits_{x\to\infty} \dfrac{1+2x-x^2}{1-x+2x^2}$
   (g) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\tan 3x}$
   (h) $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin 5x}$
   (i) $\lim\limits_{x\to 0} x\sin\dfrac{1}{x}$

(j) $\displaystyle\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

(k) $\displaystyle\lim_{x\to \pi/2}\left(\frac{\pi}{2}-x\right)\tan x$

(l) $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^x$

8. Discuss the continuity of

$$f(x)=\begin{cases}2x-x^2 & \text{if} \quad 0\le x\le 2\\ 2-x & \text{if} \quad 2<x\le 3\\ x-4 & \text{if} \quad 3<x<4\\ \pi & \text{if} \quad x\ge 4\end{cases}$$

9. Find all horizontal and vertical asymptotes of the curve $y=\dfrac{x}{\sqrt[4]{x^4+1}}$

10. Given the curve $y=\dfrac{2}{1-3x}$. Find:

    (a) the slope of the tangent line to this curve at the point $(2,1)$;
    (b) the equation of this tangent line.

11. Find $f^{(5)}(0)$ for

    (a) $f(x)=2^x$.
    (b) $f(x)=e^{2x}$.

12. What is the domain of $f(x)=\log_5(5-e^x)$?

13. Calculate y' for

    (a) $x^2y^3+3y^2=x-4y$
    (b) $\cos(x+2y)=4x^2-y^3$

14. Compute the derivative:

    (a) $y=\dfrac{(x+5)^4}{x^4+5^4}$
    (b) $y=\dfrac{1}{\sin(x-\sin x)}$
    (c) $y=\tan^5(\sqrt{1-x^2})$
    (d) $y=\ln(\cos x)$
    (e) $y=\arccos(\sqrt{t})+\arctan(5t)$

15. Suppose that $h(x)=f(x)g(x)$ and $F(x)=f(g(x))$, where $f(2)=3, g(2)=5, g'(2)=4, f'(2)=-2, f'(5)=11$. Find $h'(2)$ and $F'(2)$.

16. If $H(x)=f(x^2+4x)$ and $f'(12)=7$ find $H'(2)$.

17. Find the equation of the tangent to the curve $y=\ln(e^x+e^{2x})$ at the point $(0,\ln 2)$.

18. At what point on the curve $y=[\ln(x+4)]^2$ is the tangent line horizontal?

19. Find the linear approximation for $f(x)=\sqrt{25-x^2}$ near 3.

20. The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of the edge is 80cm.

21. A paper cup has the shape of cone with height 10cm and radius 3cm at the top. If water is poured into the cup at a rate of $2\text{cm}^3/\text{s}$, how fast is the water level rising when the water is 5cm deep?

22. A balloon is rising at a constant speed of 5ft/s. A boy is cycling along a straight road at a speed of 15ft/s. When he passed under the balloon it is 45ft above him. How fast is the distance between the boy and the balloon increasing 3s later?

23. Solve each equation for x:

    (a) $e^{e^x} = 2$
    (b) $\ln(x+1) - \ln x = 1$
    (c) $3^t = 9^{2t-1}$

24. Given $\mathbf{r}(t) = \ln t\mathbf{i} + te^{2t}\mathbf{j}$. Find parametric equations for the tangent line to the curve at the point $(0, e^2)$.

25. A bacteria culture starts with 1500 bacteria, and the population triples every 2 hours. How long will it take for the population to reach 2520 bacteria?

26. A cup of coffee has a temperature of 200°F and is in a room that has a temperature of 70°F. After 10min the temperature of coffee is 150°F. When will the coffee have cooled to 100°F?

27. If $f(x) = 3x^4 - 4x^3 - 12x^2 + 2$ find the intervals where $f(x)$ is increasing or decreasing and locate all local extrema.

28. Where is $f(x) = x \ln x$ concave up?

29. Find the absolute extreme values for $f(x) = x^3 - 12x + 5$ over the interval $[-5, 1]$.

30. Find the most general antiderivative of $\dfrac{1+4x}{\sqrt{x}}$.

31. Find $f(x)$ if $f'(x) = 1 + 2\sin x - \cos x$, $f(0) = 3$.

32. Compute

    (a) $\sin(2 \arcsin \frac{3}{5})$
    (b) $\arcsin(\sin \frac{5\pi}{4})$

33. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

34. Evaluate:

    (a) $\displaystyle\int_0^{\pi/2} \frac{\mathrm{d}}{\mathrm{d}x}\left(\sin\frac{x}{2}\cos\frac{x}{2}\right)\mathrm{d}x$
    (b) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_x^{\pi/2} \sin\frac{t}{2}\cos\frac{t}{2}\,\mathrm{d}t\right)$

35. Evaluate the integral if it exists:

    (a) $\displaystyle\int_1^8 \sqrt[3]{x}(x-1)\,\mathrm{d}x$

(b) $\int_0^b (x^3 + 4x - 1)\,\mathrm{d}x$

(c) $\int_1^4 \frac{x^2 - x + 1}{\sqrt{x}}\,\mathrm{d}x$

(d) $\int_{-1}^2 (x - 2|x|)\,\mathrm{d}x$

36. Find the area under the curve $y = 8e^x$ from ln(3) to ln(6).

From textbook:

1. page 146 problems 1-3
2. page 234 problems 1-6, 9-12
3. page 297 problems 2-9, 11, 12
4. page 356 problems 1,2,5-7, 9-12
5. page 416 problems 5-9, 13

- Topics which will NOT appear on the Final:
  - Vector Projections (1.2)
  - Orthogonal Complement (1.2)
  - Distance from point to line. (1.2)
  - Intermediate Value Theorem (2.5)
  - Graph of f' Given f (3.1)
  - Orthogonal Curves/Trajectories (3.6)
  - Unit Tangent Vectors (3.7)
  - Position/Velocity/Acceleration (3.7, 3.8)
  - Angle of Intersection of Curves (3.7)
  - Quadratic Approximation (3.11)
  - Differentials (3.11)
  - Newton's Method (3.12)
  - Showing One-to-One Function (4.2)
  - Graphical Interpretation of Function/Deriv/2nd Deriv(5.1)
  - Sketching Graph of Function Given Information about f, f', and f'' (5.1)
  - Mean Value Theorem (5.3)
  - Acceleration/Velocity/Position (5.7)
  - Antiderivatives of Vector Functions (5.7)
  - Midpoint Rule (6.3)
  - Computing a Definite Integral by the Definition (6.3)