

## Section 3.10: Related rates

In this section, we have two or more quantities that are changing with respect to time  $t$ . We will apply the following strategy:

1. Read the problem carefully and draw a diagram if possible.
2. Express the given information and the required rates in terms of derivatives and state your “find” and “when”.
3. Find a formula (equation) that relates the quantities in the problem. (If necessary, use Geometry<sup>1</sup> of the situation to eliminate one of the variables by substitution.) **Don’t substitute the given numerical information at this step!!!**
4. Use the Chain Rule to differentiate both sides of the equation with respect to  $t$ .
5. Substitute the given numerical information in the resulting equation and solve for the desired rate of change.

EXAMPLE 1. *A spherical balloon is inflated with gas at a rate of 25ft<sup>3</sup>/min. How fast is the radius changing when the radius is 2ft?*

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<sup>1</sup>Useful formulas:

- Triangle:  $A = \frac{1}{2}bh$ 
  - Equilateral Triangle:  $h = \frac{\sqrt{3}}{2}s$ ;  $A = \frac{\sqrt{3}s^2}{2}$
  - Right Triangle: Pythagorean Theorem  $c^2 = a^2 + b^2$
- Trapezoid:  $A = \frac{h}{2}(b_1 + b_2)$
- Parallelogram:  $A = bh$
- Circle:  $A = \pi r^2$ ;  $C = 2\pi r$
- Sector of Circle:  $A = \frac{1}{2}r^2\theta$ ;  $s = r\theta$
- Sphere:  $V = \frac{4}{3}\pi r^3$ ;  $A = 4\pi r^2$
- Cylinder:  $V = \pi r^2h$
- Cone:  $V = \frac{1}{3}\pi r^2h$

EXAMPLE 2. *A ladder 25 feet long and leaning against a vertical wall. The bottom of the ladder slides away from the wall at speed 3 feet/sec. Determine how fast the angle between the top of the ladder and the wall is changing when the angle is  $\frac{\pi}{4}$  radians.*

EXAMPLE 3. *A water tank has the shape of an inverted right circular cone with height 16m and base radius 4m. Water is pouring into the tank at  $3\text{m}^3/\text{min}$ .*

(a) *How fast is the water level rising when the water in the tank is 5 meters deep?*

(b) *At what rate is the radius of the top of the water in the tank changing when the depth of the water is 5 meters?*

EXAMPLE 4. *Two people are separated by 350 meters. Person A starts walking north at a rate of 0.6 m/sec and 7 minutes later Person B starts walking south at 0.5 m/sec. At what rate is the distance separating the two people changing 25 minutes after Person A starts walking?*

EXAMPLE 5. *A trough of water is 8 meters long and its ends are in the shape of isosceles triangles whose width is 5 meters and height is 2 meters. If the trough is filled with water at a constant rate of  $6\text{m}^3/\text{s}$ , how fast the water level (the height of the water) changing when the water is 120cm deep?*

EXAMPLE 6. *A plane flying with a constant speed of 360km/hour passes over a radar station at an altitude of 2km and climbs at an angle of  $30^\circ$ . At what rate is the distance from the plane to the radar station increasing 1 minute later?*