Section 3.2: Differentiation formulas

The properties and formulas in this section will be given in both “prime” notation and “fraction” notation.

PROPERTIES:

1. **Constant rule**: If f is a constant function, $f(x) = c$, then $f'(x) = 0$, or $\dfrac{\mathrm{d}c}{\mathrm{d}x} = 0$.
2. **Power rule:** If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$, or $\dfrac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$.
3. **Constant multiple rule:** If c is a constant and $f'(x)$ exists then
$$(cf(x))' = cf'(x), \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}(cf) = c\frac{\mathrm{d}f}{\mathrm{d}x}.$$
4. **Sum/Difference rule**: If $f'(x)$ and $g'(x)$ exists then
$$(f(x) + g(x))' = f'(x) + g'(x), \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}(f + g) = \frac{\mathrm{d}f}{\mathrm{d}x} + \frac{\mathrm{d}g}{\mathrm{d}x}.$$
5. **Product rule:** If $f'(x)$ and $g'(x)$ exists then
$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x), \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}(fg) = f\frac{\mathrm{d}g}{\mathrm{d}x} + g\frac{\mathrm{d}ff}{\mathrm{d}x}.$$
6. **Quotient rule:** If $f'(x)$ and $g'(x)$ exists then
$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}, \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{[g(x)]^2}.$$

EXAMPLE 1. *Find the derivatives of the following functions:*

(a) $f(x) = x^{10} + 3x^5 - 12x + 44 - \pi^5$

(b) $g(t) = (1 + \sqrt{t})^2$

(c) $F(s) = \left(\dfrac{s}{3}\right)^4 - s^{-5}$

(d) $y = \dfrac{u^5 + 1}{u^2\sqrt{u}}$

(e) $f(x) = (x^4 - 3x^2 + 11)(3x^3 - 5x^2 + 22)$

(f) $g(z) = \dfrac{4 - z^2}{4 + z^2}$

EXAMPLE 2. *The functions f and g satisfy the properties as shown in the table below:*

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-5	8	5	12
3	1	2	-2	8

Find the indicated quantity:

(a) $h'(3)$ *if* $h(x) = (3x^2 + 1)g(x)$

(b) $H'(1)$ *if* $H(x) = \dfrac{x^2}{f(x)}$

EXAMPLE 3. *Given* $f(x) = x^3 - 5x^2 + 6x - 3$

(a) *Find the equation of the tangent line to the graph of* $f(x)$ *at the point* $(1, -1)$.

(b) *Find the value(s) of* x *where* $f(x)$ *has a tangent line that is parallel to* $y = 6x + 1$.

EXAMPLE 4. *Show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$ and find their equations.*

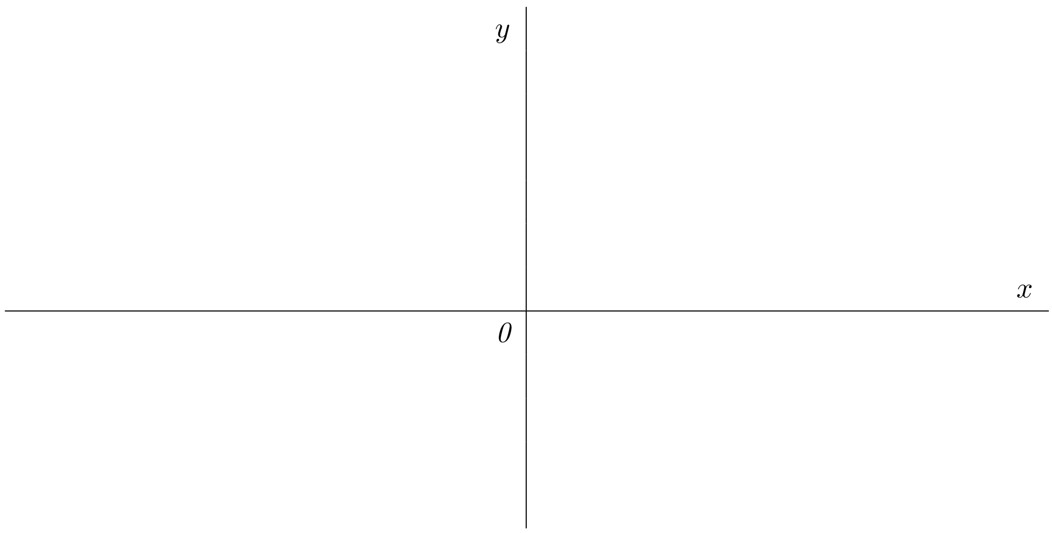
EXAMPLE 5. *Let* $f(x) = \begin{cases} -1 - 2x & \text{if} \quad x < -1 \\ x^2 & \text{if} \quad -1 \le x \le 1 \\ x & \text{if} \quad x > 1 \end{cases}$

(a) *Give a formula for f'.*

(b) *For what value(s) of x the function is not differentiable?*

(c) *Sketch the graph of f and f' on the same axis.*

.



EXAMPLE 6. *A ball is thrown into the air. Its position at time t is given by*

$$\mathbf{s}(t) = \left\langle 2t, 10t - t^2 \right\rangle .$$

(a) *Find the velocity of the ball at time $t = 2$.*

(b) *Find the speed of the ball at time $t = 2$.*

EXAMPLE 7. *Let*

$$f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ ax + b & \text{if } x > 2 \end{cases}$$

Find the values of a and b that makes f differentiable everywhere.