5.5: Applied Maximum and Minimum Problems

OPTIMIZATION PROBLEMS

First derivative test for absolute extrema: Suppose that c is a critical number of a continuous function f defined on an interval.

- If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f.
- If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f.

Alternatively,

- *If $f''(x) < 0$ for all x (so f is always concave downward) then the local maximum at c must be an absolute maximum.*
- *If $f''(x) > 0$ for all x (so f is always concave upward) then the local minimum at c must be an absolute minimum.*

EXAMPLE 1. *When a producer sells x items per week, he makes a profit of*

$$p(x) = 15x - 0.001x^2 - 2000.$$

How many items does he need to sell to get the maximum profit?

EXAMPLE 2. *A rectangular storage container with an open top is to have a volume of* $10\mathrm{m}^3$. *The length of its base is twice the width. Material for the base costs* \$10 *per squire meter. Material for the sides costs* \$6 *per square meter. Find the cost of materials for the cheapest such container.*

EXAMPLE 3. *Find the shortest distance from the parabola* $y^2 = 2x$ *to the point* $(2, 0)$.

EXAMPLE 4. *A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{9 - x^2}$. What length and width should the rectangle have so that its area is a maximum? (*Equivalently,find the dimensions of the largest rectangle that can be inscribed in the semi-disk with radius 3. *)*