Section 5.7: Antiderivatives

DEFINITION 1. *A function F is called an* **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all x in I.*

EXAMPLE 2. **(a)** *Is the function $F(x) = x\ln(x) - x + \sin x$ is an antiderivative of $f(x) = \ln(x) + \cos x$?*

(b) *Is the function $F(x) = x\ln(x) - x + \sin x + 10$ is an antiderivative of $f(x) = \ln(x) + \cos x$?*

(c) *What is the most general antiderivative of $f(x) = \ln(x) + \cos x$?*

THEOREM 3. *If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$, where C is an arbitrary constant.*

EXAMPLE 4. *Find the most general antiderivative of $f = 2x$.*

Table of Antidifferentiation Formulas

Function	Particular antiderivative	Most general antiderivative
$k \quad (k \in \mathbb{R})$	kx	$kx + C$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	
e^x	e^x	
$\cos x$	$\sin x$	
$\sin x$	$-\cos x$	
$\sec^2 x$	$\tan x$	
$\csc^2 x$	$-\cot x$	
$\sec x \tan x$	$\sec x$	
$\csc x \cot x$	$\csc x$	
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x$	
$\dfrac{1}{1+x^2}$	$\arctan x$	

EXAMPLE 5. *Find the most general antiderivative of f where*

(a) $f(x) = 5\sin x + 7x^6 - \sqrt[8]{x^7} + 15$

(b) $f(x) = \dfrac{3x + 8 - x^2}{x^3}$

(c) $f(x) = e^x + (1 - x^2)^{-1/2}$

EXAMPLE 6. *Find $f(x)$ given that $f'(x) = 4 - 3(1 + x^2)^{-1}$, $\quad f(1) = 0$.*

EXAMPLE 7. *Find* $f(x)$ *given that* $f''(x) = 3e^x + 5sinx, \quad f(0) = 1, \quad f'(0) = 2.$

EXAMPLE 8. *A particle is moving according to acceleration* $a(t) = 2t + 2$. *Find the position,* $s(t)$, *of the object at time* t *if we know* $s(0) = 1$ *and* $v(0) = -2$.

EXAMPLE 9. *Show that for motion in a straight line with a constant acceleration* a*, initial velocity* v_0*, and initial displacement* s_0*, the displacement after time* t *is*

$$s(t) = \frac{1}{2}at^2 + v_0 t + s_0.$$

EXAMPLE 10. *A stone is thrown downward from a 450 m tall building at a speed of 5 meters per second. Derive a formula for the distance of the stone above ground level.*

EXAMPLE 11. *A car braked with a constant deceleration of* $40\mathrm{ft/s}^2$*, producing skid marks measuring* $160\mathrm{ft}$ *before coming to a stop. How fast was the car traveling when the brakes were first applied?*