

Section 3.3: Rates Of Change In The Natural And Social Sciences.

Let $s(t)$ be the position function of an object. Its rate of change with respect to time is the velocity:

$$v(t) = s'(t).$$

- If $v(t) = 0$ then the object is at rest;
- if $v(t) > 0$ then the object is moving in the positive direction (i.e. is advancing, up or right);
- if $v(t) < 0$ then the object is moving in the negative direction (i.e. is retreating, down or left);

Rectilinear motion (motion along a line): A particle representing some object is allowed to move in either direction along a line.

EXAMPLE 1. *A particle is moving in a straight line. Its position is given by*

$$s(t) = 4t^3 - 9t^2 + 6t + 2,$$

where t is measured in seconds and s is measured in meters.

(a) *Find the velocity $v(t)$ of the particle at time t .*

(b) *When is the particle at rest?*

(c) *When is the particle moving in the positive direction?*

(d) Draw a diagram to represent the motion of the particle.

(e) Find the total distance the particle traveled during the first three seconds. (Hint: Calculate each distance between turns and then add to get the total.)

EXAMPLE 2. A ball is thrown vertically upward with a velocity of 80ft/s. Its height after t seconds is given by

$$s(t) = 80t - 16t^2.$$

What is the maximum height reached by the ball?

EXAMPLE 3. A spherical balloon is being inflated. Find the rate of increase of the volume with respect to the radius r when r is 1ft. (Recall that the volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)