

## 11.1: Three-dimensional Coordinate System

The three-dimensional coordinate system consists of the **origin**  $O$  and the **coordinate axes**:  $x$ -axis,  $y$ -axis,  $z$ -axis. The coordinate axes determine 3 **coordinate planes**: the  $xy$ -plane, the  $xz$ -plane and  $yz$ -plane. The coordinate planes divide space into 8 parts, called octants.

Representation of point  $P(a, b, c)$  and its projections on the coordinate planes:

- **Distance formula in  $\mathbb{R}^3$** : The distance between the points  $P(x_1, y_1, z_1)$  and  $Q(x_2, y_2, z_2)$  is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

EXAMPLE 1. Find an equation of a sphere with radius  $r$  and center

- (a)  $O(0, 0, 0)$ ;

(b)  $P(a, b, c)$ .

EXAMPLE 2. Show that the equation  $x^2 + y^2 + z^2 + x - 2y + 6z - 2 = 0$  represents a sphere, and find its center and radius.