6.4: The fundamental Theorem of Calculus

The fundamental Theorem of Calculus :

PART I If $f(x)$ is continuous on $[a,b]$ then $g(x) = \int_a^x f(t)\,\mathrm{d}t$ is continuous on $[a,b]$ and differentiable on (a,b) and $g'(x) = f(x)$.

EXAMPLE 1. *Differentiate* $g(x)$ *if*

(a) $g(x) = \int_{-4}^{x^3} e^{2t}\cos^2(1-5t)\,\mathrm{d}t$

(b) $g(x) = \int_{e^{x^2}}^{1} \frac{t+1}{\ln t + 3}\,\mathrm{d}t$

(c) $g(x) = \int_{x^2}^{\sin x} \frac{\cos t}{t}\,\mathrm{d}t$

PART II If $f(x)$ is continuous on $[a,b]$ and $F(x)$ is any antiderivative fort $f(x)$ then

$$\int_a^b f(x)\,\mathrm{d}x = F(x)\,\Big|_a^b = F(b) - F(a).$$

DEFINITION 2. *A function F is called an* **antiderivative** *of f on an interval I if $F'(x) = f(x)$ for all x in I.*

Table of Antidifferentiation Formulas

Function	Particular antiderivative	Most general antiderivative
$k \quad (k \in \mathbb{R})$	kx	$kx + C$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\dfrac{x^{n+1}}{n+1} + C$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$	
e^x	e^x	
$\cos x$	$\sin x$	
$\sin x$	$-\cos x$	
$\sec^2 x$	$\tan x$	
$\csc^2 x$	$-\cot x$	
$\sec x \tan x$	$\sec x$	
$\csc x \cot x$	$\csc x$	
$\dfrac{1}{\sqrt{1-x^2}}$	$\arcsin x$	
$\dfrac{1}{1+x^2}$	$\arctan x$	

EXAMPLE 3. *Evaluate*

(a) $\displaystyle\int_{\ln 2}^{\ln 10} \frac{1}{2} e^x \, \mathrm{d}x$

(b) $\displaystyle\int_{-1}^{5} \frac{1}{x^2} \, \mathrm{d}x$

(c) $\displaystyle\int_1^2 \frac{2x^5 - x + 3}{x^2}\,\mathrm{d}x$

(d) $\displaystyle\int_0^3 |3t-5|\;\mathrm{d}t$

Integration of symmetric functions:
Even function $f(x)$:

Examples:

$$\int_{-a}^{a} f(x)\,\mathrm{d}x =$$

Odd function $f(x)$:

Examples:

$$\int_{-a}^{a} f(x)\,\mathrm{d}x =$$

EXAMPLE 4. *Evaluate*

(a) $\int_{-2}^{2} (4x^4 + x^2 + x^5 + \sin x + e^x)\,\mathrm{d}x$

(b) $\int_{0}^{2\pi} (\cos x - 4\sin x)\,\mathrm{d}x$