7.4: Work

PROBLEM: *Find the amount of work that is done by a force in moving an object.*

Solution:

• Case 1: constant force.

Work W done in moving an object a distance d meters is given by

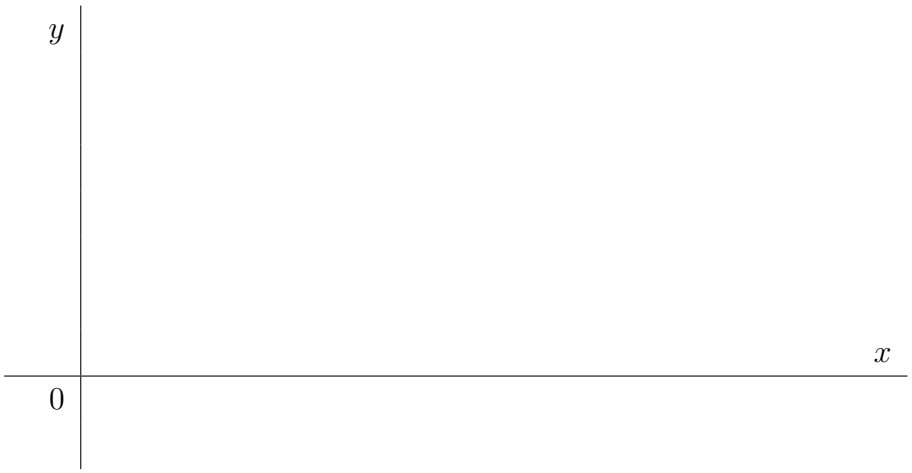
$$W = Fd.$$

In the SI metric system: $[J] = [N][m]$

In the British engineering system: [ft][lb]. Also $1 ft\text{-}lb \approx 1.36 J$.

EXAMPLE 1. *How much work is done in lifting your Calculus book (2.1kg) off the floor to put it on a desk that is* $0.6m$ *high.*

• Case 2: non constant force. (It requires integration.)



$$W_k = F(x_k^*)(x_k - x_{k-1}) = F(x_k^*)\Delta x_k$$

$$W \approx \sum_{k=1}^{n} W_k = \sum_{k=1}^{n} F(x_k^*)\Delta x_k$$

Finally, $W = \lim_{\|P\|\to 0} \sum_{k=1}^{n} F(x_k^*)\Delta x_k$ where $\|P\| = \max_k \Delta x_k$. Thus, *work done in moving an object from* $x = a$ *to* $x = b$ *is*

$$W = \int_a^b F(x)\,\mathrm{d}x$$

EXAMPLE 2. *When a particle is at distance* x *feet from the origin, a force of* $3x^2 + 2x$ *pounds acts on it. How much work is done in moving it from* $x = 1$ *to* $x = 3$ *along the* x*-axis?*

EXAMPLE 3. *A spring has a natural length of* $1m$. *If a* $50N$ *force is required to keep it stretched to a length* $3m$, *how much work is done in stretching the spring from* $2m$ *to* $5m$?

Solution By Hooke's law the force required to stretch a spring x units **beyond** its natural length is

EXAMPLE 4. *If the work required to stretch a spring* $1ft$ *beyond its natural length is* $12ft$-lb, *how much work is needed to stretch it* 9 *inches beyond its natural length?*

EXAMPLE 5. *An aquarium has a form of a cube whose side is* $1m$. *If the aquarium is full of water, find the work needed to pump* 50% *of the water out of the aquarium. (The density of water is* $1000kg/m^3$.*)*

EXAMPLE 6. *Work the previous example assuming the aquarium is only* $1/2$ *full.*

EXAMPLE 7. *A tank has a shape of an inverted circular cone with height* $10m$ *and base radius* $5m$*. It is filled with water to a height of* $8m$*. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is* $1000kg/m^3$*.)*

EXAMPLE 8. *A heavy rope* $40ft$ *long, weighs* $0.4lb/ft$ *and hangs over the edge of a building* $120ft$ *high. How much work is done in pulling the rope to the top of the building?*

EXAMPLE 9. *A uniform cable hanging over the edge of a tall building is* $20ft$ *long and weight* $30lb$*. How much work is required to pull* $5ft$ *of the cable to the top?*

REMARK 10. The exact height of the building doesn't matter.