8.1: Integration by Parts

- $\displaystyle\int x^n e^{kx}\,\mathrm{d}x$

- $\displaystyle\int x^n \sin(kx)\,\mathrm{d}x,\ \int x^n \cos(kx)\,\mathrm{d}x$

- $\displaystyle\int x^n \ln x\,\mathrm{d}x$

- $\displaystyle\int e^x \cos x\,\mathrm{d}x,\ \int e^x \sin x\,\mathrm{d}x$

- $\displaystyle\int \sec^n x\,\mathrm{d}x$ and some integrals involving inverse trigonometric functions.

The **integration by parts formula**:

$$\int fg'\,\mathrm{d}x = fg - \int f'g\,\mathrm{d}x$$

Proof:

Rewrite the above formula using the following substitutions:

$$u = f(x), \qquad v = g(x)$$

EXAMPLE 1. *Evaluate* $I = \int x \cos(5x)\, \mathrm{d}x$

EXAMPLE 2. *Evaluate* $I = \int xe^{kx}\, \mathrm{d}x$

Integration by parts formula for definite integrals:

$$\int_a^b u\, \mathrm{d}v = uv\big|_a^b - \int_a^b v\, \mathrm{d}u,$$

where

$$uv\big|_a^b = u(b)v(b) - u(a)v(a).$$

EXAMPLE 3. *Evaluate* $I = \int_{-1}^3 xe^{3x}\, \mathrm{d}x$

EXAMPLE 4. *Evaluate* $I = \displaystyle\int x^2 \sin(5x)\,\mathrm{d}x$

EXAMPLE 5. *Evaluate* $I = \displaystyle\int \ln x\,\mathrm{d}x$

EXAMPLE 6. *Evaluate* $I = \displaystyle\int x^9 \ln x\,\mathrm{d}x$

EXAMPLE 7. *Evaluate $I = \int \arcsin x \, \mathrm{d}x$*

EXAMPLE 8. *Evaluate $I = \int_0^1 \arctan x \, \mathrm{d}x$*

EXAMPLE 9. *Evaluate* $I = \displaystyle\int e^x \cos x \, \mathrm{d}x$

EXAMPLE 10. *A particle that moves along a straight line has velocity $v(t) = t^2 e^{-t}$ meters per second after t seconds. How far will it travel during the first 10 seconds?*