Fall 2012 Math 152

Week in Review 9

courtesy: *Oksana Shatalov*

(covering Sections 10.5& 10.6)

10.5: Power Series

Key Points

- For a given power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only 3 possibilities:
  1. There is $R>0$ such that the series converges if $|x-a|<R$ and diverges if $|x-a|>R$. We call such R the **radius of convergence.**
  2. The series converges only for $x=a$ (then $R=0$).
  3. The series converges for all x (then $R=\infty$).
- We find the radius of convergence using the **Ratio Test.**
- An **interval of convergence** is the interval of all x's for which the power series converges.
- You must check the endpoints $x=a\pm R$ individually to determine whether or not they are in the interval of convergence.

Examples

1. For the following series find the radius and interval of convergence.

   (a) $\sum_{n=0}^{\infty} \frac{n^4 x^n}{7^n}$

   (b) $\sum_{n=0}^{\infty} \frac{8^n (x+4)^{3n}}{n^3+1}$

   (c) $\sum_{n=1}^{\infty} \frac{(-9)^n (5x-3)^n}{n}$

   (d) $\sum_{n=1}^{\infty} \frac{(n+1)!(x-1)^{n+1}}{4^{n+1}}$

   (e) $\sum_{n=0}^{\infty} \frac{(-6)^n x^n}{(3n+1)!}$

2. Assume that it is known that the series $\sum_{n=0}^{\infty} c_n(x-3)^n$ converges when $x=5$ and diverges when $x=-2$. What can be said about the convergence or divergence of the following series:

   (a) $\sum_{n=0}^{\infty} c_n(-7)^n$

   (b) $\sum_{n=0}^{\infty} c_n 5^n$

(c) $\sum_{n=0}^{\infty} c_n(-3)^n$

(d) $\sum_{n=0}^{\infty} c_n 3^n$

(e) $\sum_{n=0}^{\infty} c_n(-1)^n$

10.6: Representation of Functions as Power Series

Key Points

- Geometric Series Formula:

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} x^{n-1} = \sum_{n=0}^{\infty} x^n, \quad -1 < x < 1.$$

- **Term-by-term Differentiation and Integration of power series:**

  If $\sum_{n=0}^{\infty} c_n(x-a)^n$ has radius of convergence $R > 0$, then $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ is differentiable (and therefore continuous) on the interval $(a-R, a+R)$ and

  - $f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$
  - $\int f(x)\,\mathrm{d}x = C + \sum_{n=0}^{\infty} \frac{c_n}{n+1}(x-a)^{n+1}$

  The radii of convergence of the power series for $f'(x)$ and $\int f(x)\,\mathrm{d}x$ are both R.

3. Find a power series representation for the following functions and determine the interval of convergence.

(a) $f(x) = \frac{4}{1+x}$

(b) $f(x) = \frac{4}{2+4x}$

(c) $f(x) = \frac{-9}{9-x^4}$

(d) $f(x) = \frac{x^{2012}}{2012-x}$

(e) $f(x) = \ln(3x+5)$

(f) $f(x) = x^5 \ln(3x+5)$

(g) $f(x) = \frac{x^4}{(1-4x)^2}$

(h) $f(x) = \arctan(16x^4)$

4. Express the integral $\int_{-0.5}^{0} \frac{\mathrm{d}x}{1-x^7}$ as a power series.