Fall 2012 Math 152

Week in Review 7

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(covering Section10.2 & Exam 2 Review)

10.2: Series

Key Points

- Infinite series $\sum_{n=1}^{\infty} a_n$ $\quad$ ($n = 1$ for convenience, it can be anything).
- Partial sums: $s_N = \sum_{n=1}^{N} a_n$. Note $s_N = s_{N-1} + a_N$.
- If $\{s_N\}_{N=1}^{\infty}$ is convergent and $\lim_{N\to\infty} s_N = s$ exists as a real number, then the series $\sum_{n=1}^{n} a_n$ is *convergent*. The number s is called the **sum** of the series.
- Series we can sum:
  - Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad -1 < r < 1$
  - Telescoping Series
- THE TEST FOR DIVERGENCE: *If* $\lim_{n\to\infty} a_n$ *does not exist or if* $\lim_{n\to\infty} a_n \neq 0$*, then the series* $\sum_{n=1}^{\infty} a_n$ *is divergent.*
- **The Test for Divergence cannot be used to prove that a series converges. It can only show a series is divergent.**

Examples

1. Given a series whose partial sums are given by $s_n = (7n + 3)/(n + 7)$, find the general term a_n of the series and determine if the series converges or diverges. If it converges, find the sum.
2. Find the sum of the following series or show they are divergent:

   (a) $\sum_{n=1}^{\infty} \frac{7+5^n}{10^n}$

   (b) $\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+3)}$
3. Write the repeating decimal $0.\overline{27}$ as a fraction.
4. Use the test for Divergence to determine whether the series diverges.

   (a) $\sum_{n=1}^{\infty} \frac{n^5}{3(n^4+3)(n+1)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6-e^{-n}}$

Exam 2 Review

1. Evaluate the integral $I = \displaystyle\int (4x^2-25)^{-3/2}\,dx$

2. Determine whether the given integral is convergent or divergent.

   (a) $\displaystyle\int_1^{\infty} \frac{4+\cos^4 x}{x}\,dx$

   (b) $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{x}+e^{4x}}\,dx$

3. Evaluate $I = \displaystyle\int_0^{2012} \frac{1}{\sqrt{2012-x}}\,dx.$

4. The curve $y=\sin x$ for $0 \le x \le \pi$ is rotated about the x-axis. Set up, *but don't evaluate* the integral for the area of the resulting surface.

5. Determine if the sequence $\{a_n\}_{n=2}^{\infty}$ is decreasing and bounded:

   (a) $a_n = \ln n$

   (b) $a_n = \cos n^2$

   (c) $a_n = e^{-n}$

   (d) $a_n = e^n + 11$

   (e) $a_n = 1 - \frac{1}{n^2}$

6. The curve $y = \frac{1}{2}(e^x + e^{-x})$, $0 \le x \le 1$, is rotated about the x-axis. Find the area of the resulting surface.

7. Set up, *but don't evaluate* the integral for the length of the curve $x = 2t^2, \quad y = t^3, \quad 0 \le t \le 1.$

8. Find length of the curve $y = \frac{1}{\pi}\ln(\sec(\pi x))$ from the point $(0,0)$ to the point $(\frac{1}{6}, \ln\frac{2}{\sqrt{3}})$.

9. Use a trigonometric substitution to eliminate the root: $\sqrt{24-12x+2x^2}$.

10. Determine if the sequence converges or diverges. If converges, find its limit.

    (a) $\displaystyle\left\{\frac{2012+(-1)^n}{n^{2012}}\right\}_{n=1}^{\infty}$

    (b) $\displaystyle\left\{\sqrt{\frac{7n+6n^3+n^2}{(n+3)(n^2+8)}}\right\}_{n=4}^{\infty}.$

11. Evaluate the integral $\displaystyle\int \frac{(x-1)^2}{5\sqrt{25-(x-1)^2}}\,dx.$

12. Compute $S = \displaystyle\sum_{n=1}^{\infty}\left(e^{1/n} - e^{1/(n+1)}\right).$

13. Write out the form of the partial fraction decomposition (do not try to solve)

$$\frac{20x^3+12x^2+x}{(x^3-x)(x^3+2x^2-3x)(x^2+x+1)(x^2+9)^2}$$

14. Evaluate the integral $\displaystyle\int \frac{5x^2+x+12}{x^3+4x}\,\mathrm{d}x$

15. Assuming that the sequence defined recursively by $a_1=1$, $a_{n+1}=\frac{1}{2}\left(a_n+\frac{16}{a_n}\right)$ is convergent, find its limit.

16. For what values of x the series $\displaystyle\sum_{n=0}^{\infty}(4x-3)^{n+3}$ converges? What is the sum of the series?