Section 3.1: Derivative

DEFINITION 1. *The* **Derivative** *of a function* $f(x)$ *at* $x = a$ *is*

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

Other common notations for the derivative of $y = f(x)$ *are* f', $\frac{\mathrm{d}}{\mathrm{d}x} f(x)$.

It follows from the definition that the derivative $f'(a)$ measures:

- The slope of the tangent line to the graph of $f(x)$ at $(a, f(a))$;
- The instantaneous rate of change of $f(x)$ at $x = a$;
- The instantaneous velocity of the object at time at $t = a$ (if $f(t)$ is the position of an object at time t).

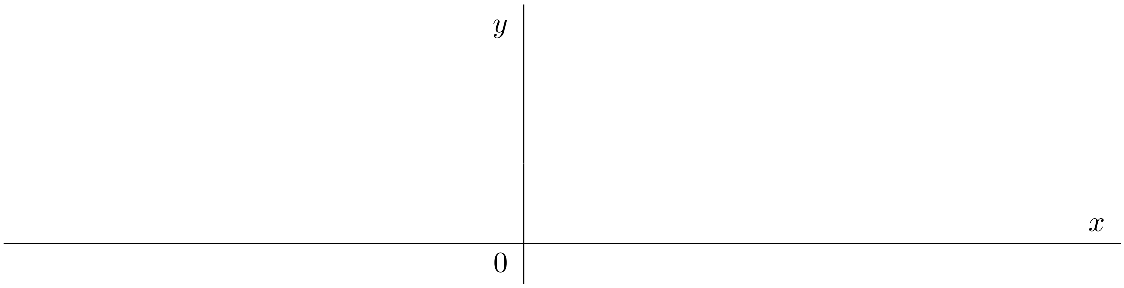
EXAMPLE 2. *Given* $f(x) = \frac{3}{x+5}$. *Use definition of the derivative to calculate* $f(x)$ *at* $x = -3$.

EXAMPLE 3. *Each limit below represents the derivative of function* $f(x)$ *at* $x = a$. *State* f *and* a *in each case.*

(a) $\lim_{h\to 0} \frac{(3+h)^4 - 81}{h}$

(b) $\lim_{x\to \frac{3\pi}{2}} \frac{\sin x + 1}{x - \frac{3\pi}{2}}$

Question: Where does a derivative not exist for a function?



DEFINITION 4. *A function $f(x)$ is said to be* **differentiable** *at $x = a$ if $f'(a)$ exists.*

EXAMPLE 5. *Refer to the graph above to determine where f(x) is not differentiable.*

CONCLUSION: A function $f(x)$ is NOT differentiable at $x = a$ if

- $f(x)$ is **not continuous** at $x = a$;

- $f(x)$ has a **sharp turn** (corner or cusp) at $x = a$ (left and right derivatives are not the same);

- $f(x)$ has a **vertical tangent** at $x = a$.

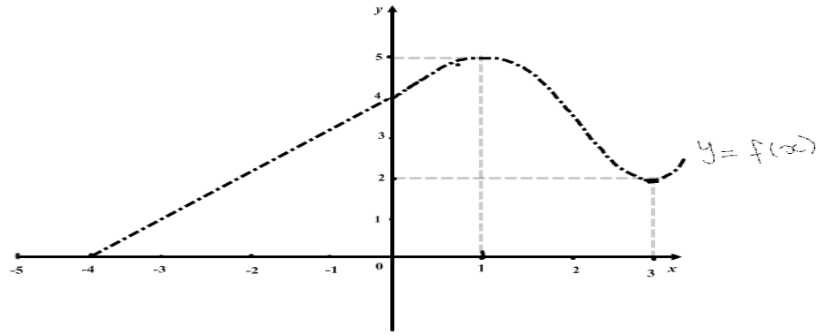
THEOREM 6. *If f is differentiable at a then f is continuous at a.*

The derivative as a function: If we replace a by x in Definition 1 we get:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

A new function $g(x) = f'(x)$ is called the **derivative of** f.

EXAMPLE 7. *Use the graph of $f(x)$ below to sketch the graph of the derivative $f'(x)$.*



EXAMPLE 8. *Use the definition of the derivative to find $f'(x)$ for $f(x) = \sqrt{1+3x}$.*