4.4: Derivatives of Logarithmic Functions

EXAMPLE 1. Using Implicit Differentiation find the derivatives of the following function:

(a)
$$f(x) = \ln x$$

(b)
$$f(x) = a^x$$

Combining the formulas obtained in Example 1 and Chain Rule one can get

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{ln}(g(x)) =$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}a^{g(x)} =$$

EXAMPLE 2. Find the derivative:

(a)
$$f(x) = \ln(\sin x)$$

(b)
$$f(x) = \ln |x|$$

(c)
$$f(x) = 5^{\cot x}$$

EXAMPLE 3. Using the change of base formula, find the derivative formula for $f(x) = \log_a x$ and generalize it using the Chain Rule.

EXAMPLE 4. Find the derivative of $f(x) = \log_2(3 + x^2 + x^3)$

Logarithmic Differentiation can be used to find derivative of complicated functions involving products, quotients or powers.

STEPS IN LOGARITHMIC DIFFERENTIATION:

- 1. Take logarithms of both sides of an equation y = f(x) and simplify (f.ex. split a product or quotient, etc.).
- 2. Differentiate implicitly with respect to x.
- 3. Solve the resulting equation for y'.
- 4. Plug in y = f(x).

EXAMPLE 5. Find the derivative: $y = (\cos x)^{\sin x}$