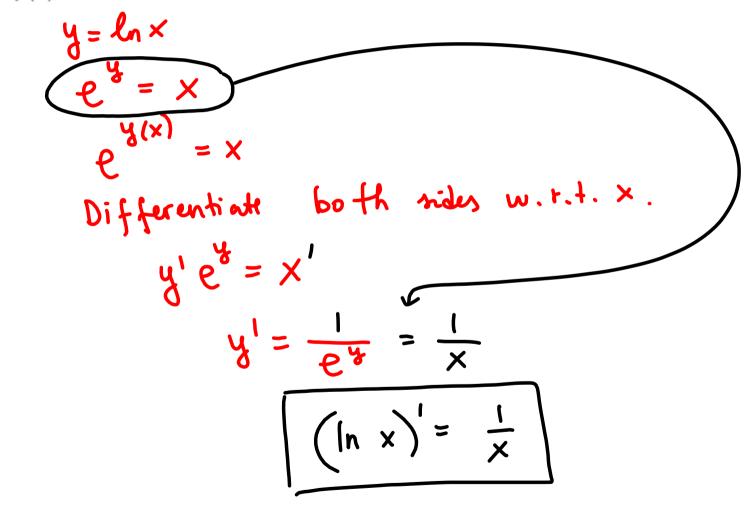
4.4: Derivatives of Logarithmic Functions

EXAMPLE 1. Using Implicit Differentiation find the derivatives of the following function:

(a)
$$f(x) = \ln x$$



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(b)
$$f(x) = a^{x}$$
 $y = a^{x}$
 $y = a^{x}$

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Combining the formulas obtained in Example 1 and Chain Rule one can get

 $\frac{\mathrm{d}}{\mathrm{d}x}\ln(g(x)) = \frac{g'(x)}{g(x)}$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}a^{g(x)} = \mathbf{a}^{g(x)} \ln \mathbf{a} \quad g'(x).$$

EXAMPLE 2. Find the derivative:

(a)
$$f(x) = \ln(\frac{\sin x}{x})$$

$$f'(x) = \frac{1}{\sin x} (\sin x)^{1} = \frac{\cos x}{\sin x} = \cot x$$

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(b)
$$f(x) = \ln |x| =$$

$$\begin{cases} \ln x, & x > 0 \\ \ln (-x), & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} (\ell_n x)^{1}, & x > 0 \\ (\ell_n (-x))^{1}, & x < 0 \end{cases} = \begin{cases} \frac{1}{x}, & x > 0 \\ \frac{1}{-x} (-1), & x < 0 \end{cases} = \frac{1}{x}$$

$$(\ell_n |x|)^{1} = \frac{1}{x}.$$

(c)
$$f(x) = 5^{\cot x}$$

$$f'(x) = 5^{\cot x} \ln 5 \quad (\cot x)'$$

$$f'(x) = 5^{\cot x} \ln 5 \quad (-\cos^2 x)$$

$$f'(x) = -(\ln 5) 5^{\cot x} \cos^2 x$$

EXAMPLE 3. Using the change of base formula, find the derivative formula for $f(x) = \log_a x$ and generalize it using the Chain Rule.

f(x)=
$$\log_a x = \frac{\ln x}{\ln a} = \frac{1}{\ln a} \ln x$$

$$f'(x) = \frac{1}{\ln a} (\ln x)' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$(\log_a x)' = \frac{1}{x \ln a} (\log_a g(x))' = \frac{g'(x)}{g(x) \ln a}$$

u=9(x)

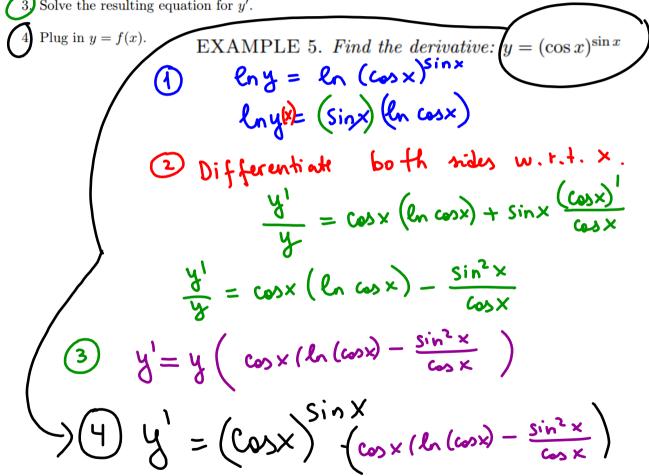
EXAMPLE 4. Find the derivative of $f(x) = \log_2(3 + x^2 + x^3)$

$$f'(x) = \frac{(3+x^2+x^3)!}{(3+x^2+x^3)! \ln 2} = \frac{2x+3x^2}{(3+x^2+x^3)! \ln 2}$$

Logarithmic Differentiation can be used to find derivative of complicated functions involving products, quotients or powers.

STEPS IN LOGARITHMIC DIFFERENTIATION:

- Take logarithms of both sides of an equation y = f(x) and simplify (f.ex. split a product or quotient, etc.).
- Differentiate implicitly with respect to x.
- Solve the resulting equation for y'.



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$$\frac{Ex}{y} = \frac{(\tan x)^{3} \sqrt{x^{2} + \cos x + 5} \sqrt{2017 - x - x \sqrt{x}}}{[\cos (\tan x^{3})]^{3}}$$

$$\ln y = \ln (\tan x) + \frac{1}{3} \ln (x^{2} + \cos x + 5) + \frac{1}{2} \ln (2017 - x - x \sqrt{x})$$

$$- 3 \ln (\cos (\tan x^{3}))$$

; y'=...