

NOTE: *The list below doesn't cover all the material and it is not sufficient to solve only these problems before the test. Review notes, suggested and webassign problems, week in review and quiz problems.*

1. Consider two planes:

$$x + y - z = 2 \tag{1}$$

and

$$3x - 4y + 5z = 6. \tag{2}$$

- (a) Find the normal vectors of these planes.

- (b) Find cosine of the angle between the planes.

- (c) Find equations for the line of intersection of these planes.

2. Consider the function

$$f(x, y) = \frac{x^2 + y^2 + 1}{x^2 + y^2 - 1}. \tag{3}$$

- (a) Find and sketch the domain of the function.

(b) Draw a contour map of the function drawing several level curves.

3. Consider the function

$$z = \ln(x - 3y). \quad (4)$$

(a) Find  $z_x(7, 2)$  and  $z_y(7, 2)$ .

(b) Find the differential of the function at the point  $(7, 2)$ .

(c) Using linear approximation approximate the value of the function at the point  $(6.9, 2.06)$ .

4. Show that

$$y \frac{\partial z}{\partial x} - x \frac{\partial z}{\partial y} = 0$$

if  $z = f(x^2 + y^2)$ .

5. Consider the following function

$$z = x^2 + y^2 - 2x + 6y + 10. \quad (5)$$

(a) Find an equation of the tangent plane to the graph of this function at the point  $(2, 0, 10)$ .

(b) Classify and sketch the graph of given function.

6. Find  $z_x$  and  $z_y$  if  $y^2ze^{x+y} - \sin(xyz) = 32$ .

7. The radius of a right circular cylinder is decreasing at a rate  $1.2\text{cm}/s$  while its height is increasing at a rate of  $3\text{cm}/s$ . At what rate the volume of the cylinder changing when the radius is  $80\text{cm}$  and the height is  $150\text{cm}$ .
8. The curve  $C$  is given by the vector function  $\mathbf{r}(t) = \langle 2 \cos t, \sin t, t \rangle$ . Find a tangent line to  $C$  at the point  $(0, 1, \pi/2)$ .
9. Find the point where the line  $x = 2 - t, y = 1 + 3t, z = 4t$  intersects the plane  $2x - y + z = 4$ .  
At this point  $x + y + z =$
- a. 9
  - b.  $-3$
  - c. 0
  - d. 3
  - e.  $-9$
10. Use the linear approximation to  $f(x, y) = \sqrt{x}e^y$  to estimate (approximate) the value  $\sqrt{0.99}e^{0.02}$
- a. 1.03
  - b. 0.975
  - c. 1.025
  - d. 1.015
  - e. 1.01
11. Given  $z = \ln(x^2 + y^2 - 9)$ .
- (a) Find and sketch the domain of the given function.

(b) Find the first partial derivatives of the given function at the point  $(3, 1)$ .

(c) Find an equation of the tangent plane to the graph of the given function at the point  $(3, 1, 0)$ . Simplify it.

12. The length and width of a rectangle are measured as  $30\text{cm}$  and  $20\text{cm}$ , respectively, with an error in measurement of at most  $0.1\text{cm}$  in each. Use differentials to estimate the maximum error in calculated area of the rectangle.

13. Let  $C$  be the curve with given vector equation:  $\mathbf{r}(t) = \langle t, \sqrt{2} \sin t, \sqrt{2} \cos t \rangle$ .

(a) Find the derivative  $\mathbf{r}'(t)$ .

(b) Find parametric equations for the tangent line to  $C$  at the point  $(\frac{\pi}{4}, 1, 1)$ .