MATH 251 Extra Practice (for Exam 2)

NOTE: *The list below doesn't cover all the material and it is not sufficient to solve only these problems before the test. Review notes, suggested and webassign problems, week in review and quiz problems.*

1. For $f(x,y,z) = x^3 + \sin(xyz)$ find the maximum rate of change of f at the point $(1, \frac{\pi}{2}, 1)$.

2. Find the equation of the plane which is tangent to the surface $ze^{xyz} = 1$ at the point $(5, 0, 1)$.

3. The equation of the plane tangent to the surface $ze^{\frac{xz}{y}} = 1$ at $(x,y,z) = (0, \frac{1}{2}, 1)$ is

   a. $\frac{1}{2}x + z - 1 = 0$

   b. $x = 0$

   c. $2x + z - 1 = 0$

   d. $2x + y + z = \frac{3}{2}$

   e. $2x - y + z = \frac{1}{2}$

4. The temperature at a point (x,y,z) is given by

$$T(x,y,z) = e^{-(x+y+z)^3}.$$

   The maximum rate of change of T at the point $P(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$ is

   a. $\sqrt{3}$

   b. e^{-1}

   c. $-3\sqrt{3}e^{-1}$

   d. $\sqrt{3}e^{-1}$

   e. $3\sqrt{3}e^{-1}$

5. For the function $f(x,y) = xy^2 + x^3 - 2xy$ the point $(x,y) = (\frac{1}{\sqrt{3}}, 1)$ is

   a. a local minimum

   b. a local maximum

   c. a saddle point

   d. not a critical point

   e. is a critical point but the Second Derivative Test fails.

6. Let

$$f(x,y) = xy - 2x + 5.$$

   Find the absolute maximum and minimum values of f on the set D which is the closed triangular region with vertices $A(0,0), B(1,1), C(0,1)$.

7. For

$$\int_0^3 \int_{y^2}^9 f(x,y)\, \mathrm{d}x\mathrm{d}y$$

(a) sketch the region of integration;

(b) change the order of integration.

8. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 4$.

9. Convert the integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz \, dz \, dx \, dy$$

to an integral in cylindrical coordinates, but don't evaluate it.

10. Find the absolute maximum and minimum values of $f(x, y) = x^2y + xy^2 + y^2 - y$ on the set D which is the closed rectangular region in the xy-plane with vertices $(0, 0)$, $(0, 2)$, $(2, 0)$ and $(2, 2)$.

11. Evaluate the integral by reversing the order of integration:

$$\int_0^3 \int_{y^2}^9 y \cos(x^2) \mathrm{dxdy}.$$

12. Find the mass of the lamina that occupies the region bounded by the parabola $x = y^2$ and the line $y = x - 2$ and has the density $\rho(x, y) = 3$.

13. Find the volume of the solid region E in the first octant bounded by the paraboloid $z = x^2 + y^2$, the cone $z = \sqrt{x^2 + y^2}$ and the coordinate planes.

14. Find the volume of the solid that lies under the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane.

15. Find the dimensions and volume of the largest rectangular solid box which sits on the xy-plane and has its upper vertices on the paraboloid $z = 4 - 4x^2 - y^2$.

16. Find the mass of the quarter circle $x^2 + y^2 \leq 9$ for $x \geq 0$ and $y \geq 0$ if the density is $\rho(x, y) = \sqrt{x^2 + y^2}$.

17. Find the center of mass of the quarter circle $x^2 + y^2 \leq 9$ for $x \geq 0$ and $y \geq 0$ if the density is $\rho(x, y) = \sqrt{x^2 + y^2}$.

18. Find the volume of the cone $z = 2\sqrt{x^2 + y^2}$ below the paraboloid $z = 8 - x^2 - y^2$.

19. Let

$$f(x, y) = 4xy^2 - x^2y^2 - xy^3.$$

Find the absolute maximum and minimum values of f on the set D which is the closed triangular region with vertices $A(0, 0), B(0, 6), C(6, 0)$.

20. Let

$$f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2).$$

Find the absolute maximum and minimum values of f on the set D which is the disk $x^2 + y^2 \leq 1$.

21. Calculate the value of the integral $\int\int_D (x^2 + y^2)^{3/2} \mathrm{dA}$, where D is the region in the first quadrant bounded by the lines $y = 0$, $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.