13.5: Double integrals in polar coordinates

EXAMPLE 1. *Evaluate*

$$I = \iint_D \arctan\frac{y}{x}\,\mathrm{d}A$$

where $D = \left\{(x,y)|\ 1 \le x^2+y^2 \le 4, x \le y \le \sqrt{3}x, x \ge 0\right\}$.

THEOREM 2. *Change to polar coordinates in a double integral: Let* f *be a continuous on the region* D*. Denote by* D^* *the region representing* D *in the polar coordinates* (r,θ)*. Then*

$$\iint_D f(x,y)\,\mathrm{d}A = \iint_{D^*} f(r\cos\theta, r\sin\theta)\ r\,\mathrm{d}r\mathrm{d}\theta.$$

REMARK 3. Be careful not to forget the additional factor r on the right side of the formula.

Solution of Example 1: Evaluate $I = \iint_D \arctan\frac{y}{x}\,\mathrm{d}A$ where $D = \left\{(x,y)|\ 1 \le x^2+y^2 \le 4, x \le y \le \sqrt{3}x, x \ge 0\right\}$.

EXAMPLE 4. *Find the volume of the solid that lies under the paraboloid* $z = x^2 + y^2$*, above the* xy*-plane and inside the cylinder* $x^2 + y^2 = 2x$.

EXAMPLE 5. *Find the area of the region inside the circle* $r = 4\sin\theta$ *and outside the circle* $r = 2$.