Complex Numbers

Equation $r^2+1=0$ *doesn't have real roots!*

Imaginary unit i : $i^2=-1$.

$$r^2+1=0 \Rightarrow r=i=\sqrt{-1}$$

Complex Numbers

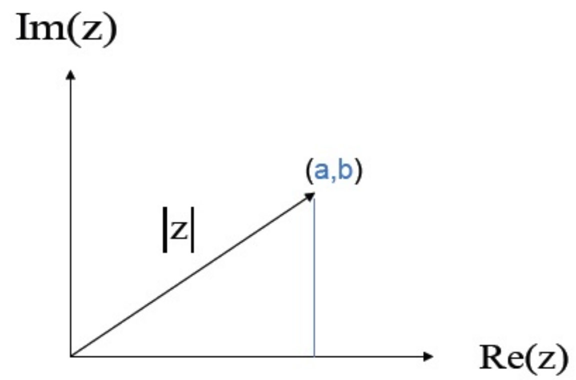
$i^2=-1$

$z=a+ib$

where a and b are real numbers (also point (a,b) in R^2)

$a=Re(z)$ real part

$b=Im(z)$ imaginary part



Modulus (or length) $|z|=\sqrt{a^2+b^2}$

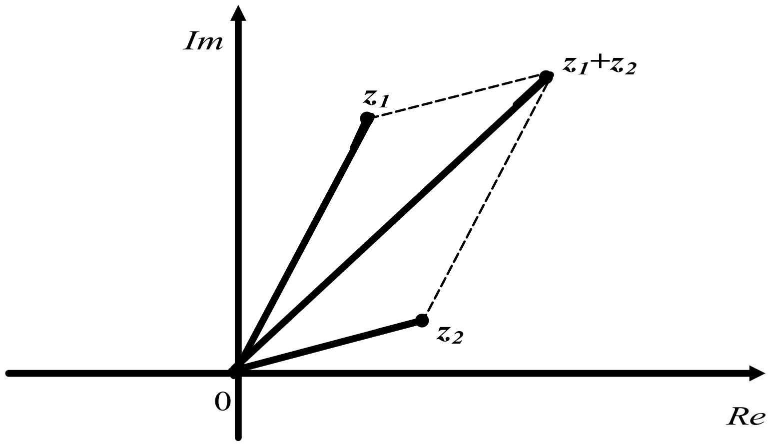
"Pure Imaginary" numbers are multiples of imaginary unit i: $z=ib$

Addition

$z_1 = a_1 + ib_1$

$$z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$$

$z_2 = a_2 + ib_2$



Multiplication (use $i^2=-1$ and FOIL)

$z_1=a_1+ib_1$

$$z_1z_2=(a_1+ib_1)(a_2+ib_2)$$

$z_2=a_2+ib_2$

$$=(a_1a_2-b_1b_2)+i(b_1a_2+a_1b_2)$$

Complex Conjugate $\bar{z}=a-ib$

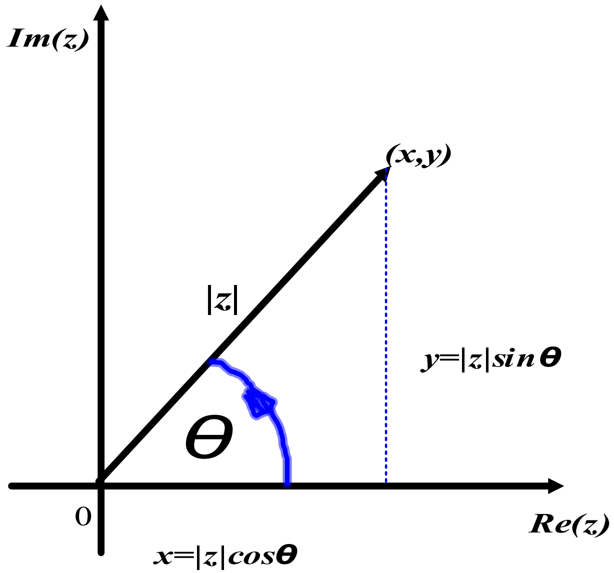
Polar form of complex numbers

$x = r\cos\theta \quad y = r\sin\theta$

Modulus: $|z| = r$

Argument : $\theta = \arg(z) = \arctan(y/x)$

(which quadrant?)

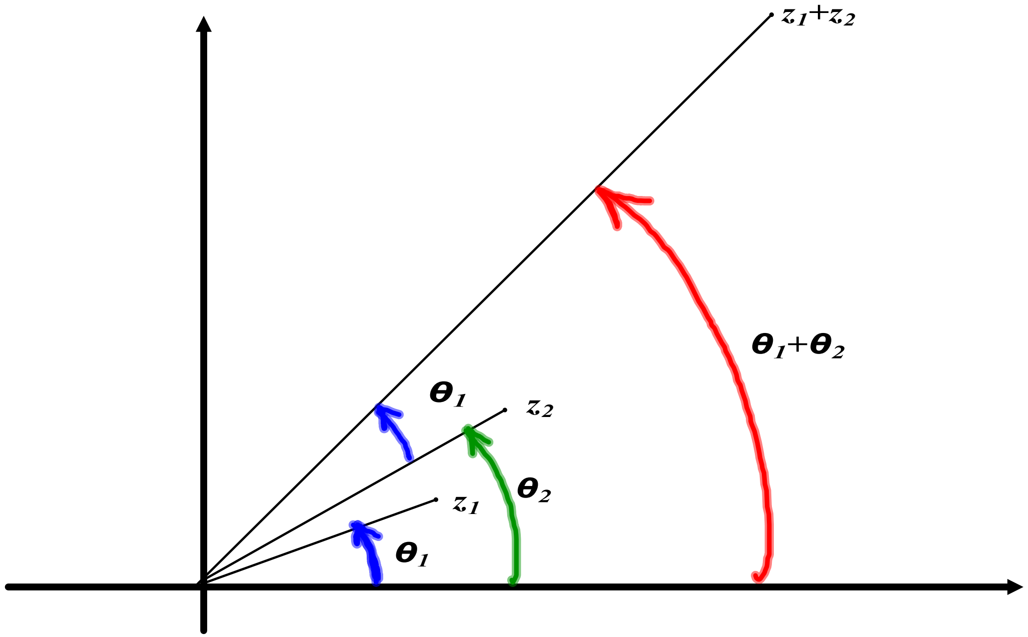


$$z = x + i\,y = r\cos\theta + i\,r\sin\theta$$

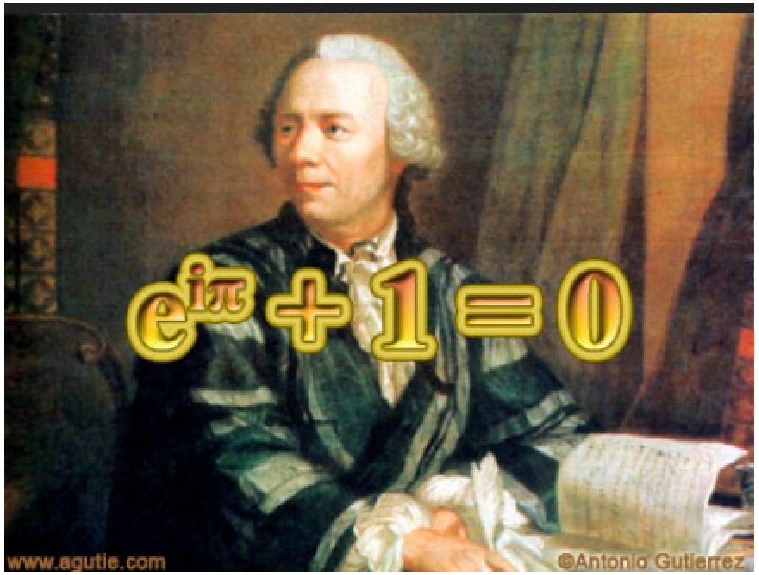
$$z = r\,(\cos\theta + i\sin\theta)$$

Properties: $|z_1 z_2| = |z_1|\ |z_2|$

$$arg(z_1 + z_2) = arg(z_1) + arg(z_2)$$



EULER notation



Recall that

$$e^x = 1+x+\frac{x^2}{2!}+\ldots+\frac{x^n}{n!}+\ldots$$

$$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots+(-1)^n\frac{x^{2n}}{(2n)!}+\ldots$$

$$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\ldots$$

Exponent of purely imaginary number ***iθ***

$$e^{i\theta} = (1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\ldots)+i\,(\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\ldots)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler formula

Exponent of a complex number $z=x+iy$

$$e^{z}=e^{x+iy}=e^{x}(\cos y+i\sin y)$$