3. Linear Equations: Method of Integrating Factors (2.1)

1. A first order ODE is called **linear** if it is expressible in the form

$$y' + p(t)y = g(t) \tag{1}$$

where $p(t)$ and $g(t)$ are given functions.

2. If $g(t) \equiv 0$ then

$$y' + p(t)y = 0 \tag{2}$$

is called a **homogeneous** liner ODE. Otherwise (1) is called a **non-homogeneous** liner ODE.

3. The equation (1) is separable if and only if there is a real constant α such that $g(t) = \alpha p(t)$. (We already know how to solve it!)

4. The method to solve (1) for arbitrary $p(t)$ and $q(t)$ is called

The Method of Integrating Factors

Step 1 Put ODE in the form (1).

Step 2 Find the integrating factor

$$\mu(t) = e^{\int p(t)\mathrm{d}t}$$

Note: Any μ will suffice here, thus take the constant of integration $C = 0$.

Step 3 Multiply both sides of (1) by μ and use the Product Rule for the left side to express the result as

$$(\mu(t)y(t))' = \mu g(x) \tag{3}$$

Step 4 Integrate both sides of (3). Note: Be sure to include the constant of integration in this step!

Step 5 Solve for the solution $y(t)$.

5. Consider

$$y' - 3xy = -xe^{x^2}.$$

(a) Find the general solution.

(b) Find the solution satisfying the initial condition $y(0) = y_0$.

(c) How do the solutions behave as x becomes large (i.e $x \to +\infty$)? Does that behavior depend on the choice of the initial value y_0?