

## 26: Homogeneous Linear Systems with Constant Coefficients (sec. 7.5)

1. **Proposition.** *If  $\lambda$  is an eigenvalue of matrix  $A$  and  $v$  is an eigenvector corresponding to this eigenvalue then*

$$X(t) = e^{\lambda t}v$$

*is a solution of the system  $X' = AX$ , i.e solution of the homogeneous linear system with constant coefficients.*

### Real Distinct Eigenvalues

2. **FACT:** *If  $A$  has  $n$  distinct real eigenvalues  $\lambda_1, \dots, \lambda_n$  and  $v_1, \dots, v_n$  are the corresponding eigenvectors, then*

$$\{e^{\lambda_1 t}v_1, \dots, e^{\lambda_n t}v_n\}$$

*is a fundamental set of solutions and the general solution is*

$$X(t) = C_1 e^{\lambda_1 t}v_1 + \dots + C_n e^{\lambda_n t}v_n.$$

Note that here the only thing should be justified is why the Wronskian is not equal to zero, which can be done by induction.

3. **EXAMPLE.** Consider the following system of DE:

$$\begin{aligned} x_1' &= -2x_1 + x_2 \\ x_2' &= 2x_1 - 3x_2 \end{aligned} \tag{1}$$

- (a) Find general solution of (1).

- (b) Find solution of (1) subject to the initial condition  $X(0) = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

- (c) What is behavior of the solution as  $t \rightarrow +\infty$ ?

4. **Question.** If  $A$  is  $2 \times 2$  and has two distinct eigenvalues  $\lambda_1, \lambda_2$  and  $v_1, v_2$  are the corresponding eigenvectors then the general solution is

$$X(t) = C_1 e^{\lambda_1 t}v_1 + C_2 e^{\lambda_2 t}v_2.$$

For what  $C_1, C_2$  the general solution tends to zero as  $t$  increases?

5. **EXAMPLE.** Find general solution of the system

$$\begin{aligned} x_1' &= x_1 - x_2 + 4x_3 \\ x_2' &= 3x_1 + 2x_2 - x_3 \\ x_3' &= 2x_1 + x_2 - x_3 \end{aligned}$$