WORKSHEET (Extra Credit +20 points to Homework 4)

Directions: Solve Examples 1,2. For Example 2 represent a solution supported by FACTS 1-4 below. **Due Wednesday February 6** at the beginning of class. Staple this extra credit behind the Homework 4 solution.

EXAMPLE 1. *Find roots of the following quadratic equations:*

(a) $4r^2 - 12r + 9 = 0$

(b) $r^2 - 5 = 0$

(c) $4r^2 + r - 1 = 0$

(d) $r^2 + r + 1 = 0$

Facts from Algebra

- FACT 1: **Cramer's Rule** for solving the system of equations

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_1x + b_2y &= c_2 \end{aligned}$$

The rule says is that if the `determinant of the coefficient matrix is not zero`, i.e.

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0,$$

then the system has a unique solution (x, y) given by

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, \qquad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

- FACT 2: If determinant of the coefficient matrix is zero then either there is no solution, or there are infinitely many solutions.

- FACT 3. The homogeneous system of linear equations

$$\begin{aligned} a_1x + b_1y &= 0 \\ a_1x + b_2y &= 0 \end{aligned}$$

always has the "trivial" solution $(x, y) = (0, 0)$. By Cramer's rule this is the only solution if the determinant of the coefficient matrix is not zero.

- FACT 4: If determinant of the coefficient matrix of homogeneous system of linear equations is zero then there are infinitely many nontrivial solutions $(x, y) \neq (0, 0)$.

EXAMPLE 2. *Use the above Facts 1-4 to solve the following system of linear equations:*

(a) $$\begin{aligned} 2x + 3y &= 10 \\ 3x - 5y &= 4 \end{aligned}$$

(b) $$\begin{aligned} 2x - 2y &= 4 \\ x - y &= 7 \end{aligned}$$

(c) $$\begin{aligned} 2x - 2y &= 0 \\ 3x - 3y &= 0 \end{aligned}$$