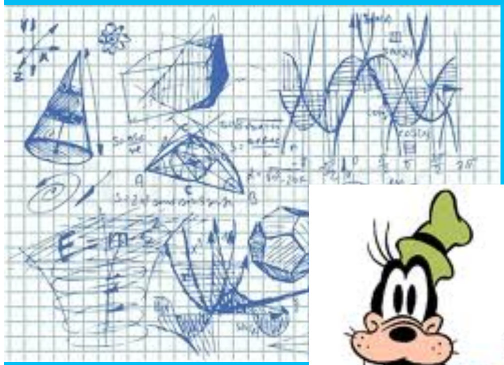
VECTOR FUNCTIONS IN THE WONDERFUL WORLD OF DISNEY!

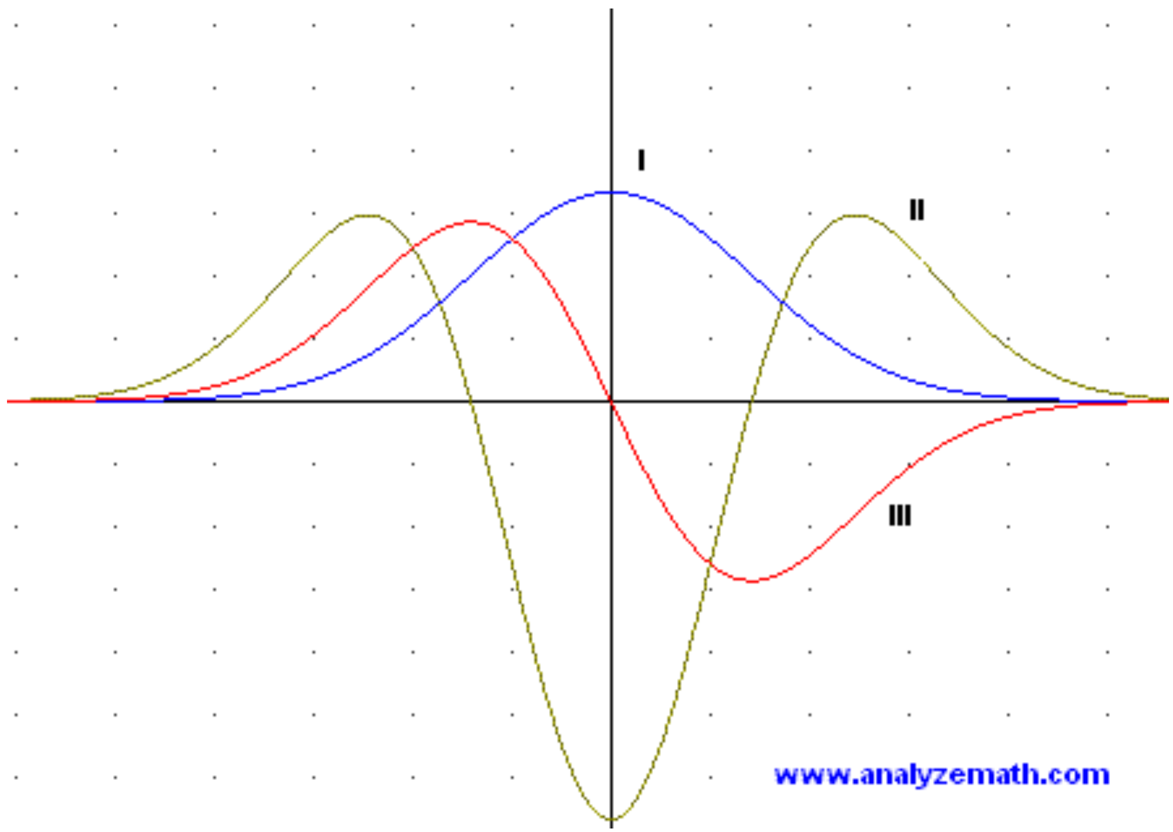
- Presented by:
- David Heath, David Heggland, Zoe Koczo, and Kristina Klock



DEFINITION

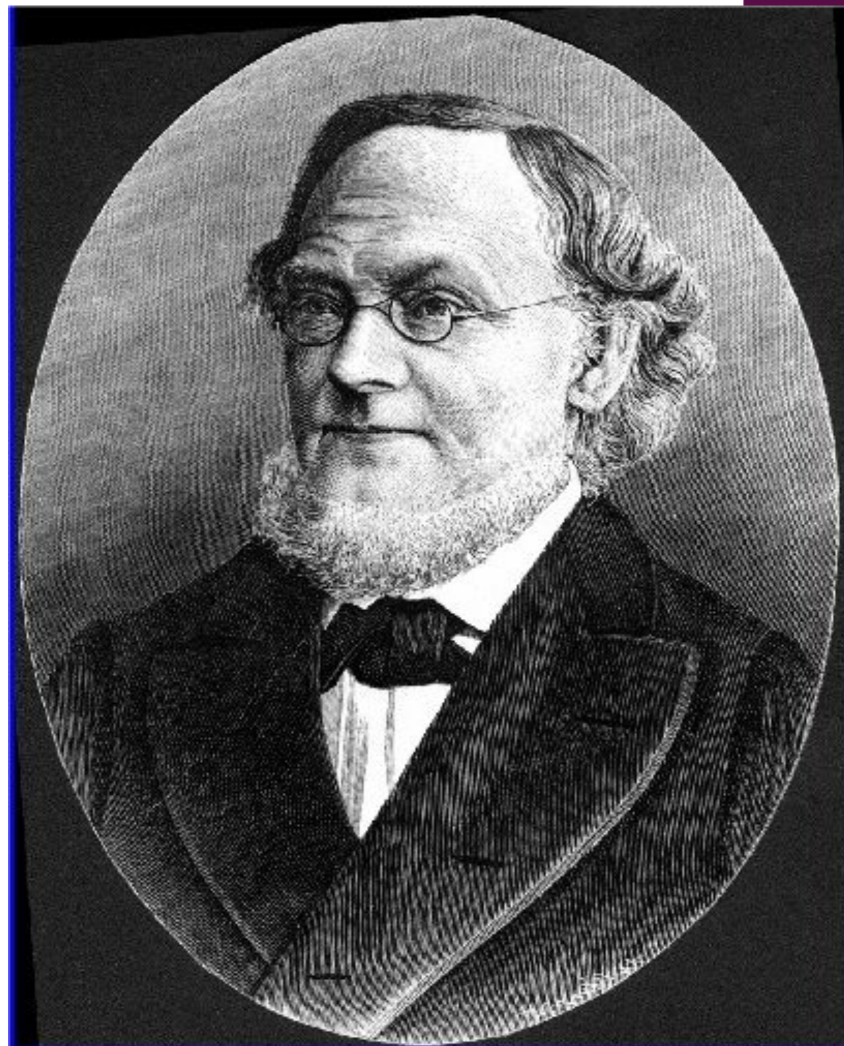
- A vector function is a function whose domain is a set of real numbers and range is a set of vectors, denoted by **r(t)** = <f(t), g(t)>.
- The parameter t (time) is inputted into the component functions of f and g to yield a real number output. Vectors are equal if they have the same magnitude and direction.
- The position of any point at time t on a vector can be represented by **r**(t); velocity **r**’(t); speed abs(**r**’(t)) because speed is the magnitude of velocity; and acceleration **r**’’(t).
- Vectors relate to life because they come in handy anytime one has an object in motion, such as an airplane or car. One can calculate the speed, velocity, acceleration, and position with one simple equation.

These functions represent position (I), velocity (III), and acceleration (II). Position is the original function, while velocity is the first derivative and acceleration the second. With vectors, we can represent position, velocity, and acceleration in component vectors.



THE FATHER OF VECTOR FUNCTIONS

- Grassmann, yet another unappreciated genius during his time, is said to have published and developed the notion of a vector space and linear algebra
- His work, *The Theory of Linear Extension, A New Branch of Mathematics*, essentially founded vector space mathematics
- He submitted this work as a PhD thesis, but it was seen as too radical and was rejected
- Grassmannian is a space named after Grassmann that parameterizes all linear substances of a vector space V of a given dimension.



THE DISNEY POLICE

- The Disney World Police are responding to a call reported by a concerned Goofy, apparently they've seen a suspicious 40 year old man dressed in a lime t-shirt and he's armed! Find the equation of the route they will take that passes by the Haunted Mansion and runs perpendicular with the east gate entrance, and then find the time it takes for the police men to arrive after they pass the haunted house.

SOLUTION:

- GIVEN:
  - Haunted Mansion @ point (X,Y) =(7,3)
  - East Gate Entrance Road @ Vector = 2Ti+5Tj,   Where T = Time
  - Location of suspect @ point (X,Y) = (12,1)
- SOLUTION:
  - Find perpendicular vector
    - Vector = 5Ti-2Tj
  - Given the points of the haunted mansion, and the perpendicular vector create two equations
    - X=7+5T,   Y=3-2T
  - Using the points of the subject calculate the time
    - 12=7+5T,   1=3-2T
    - 5=5T,   -2=-2T
    - 1=T,   1=T

It takes the police men 1 minute to reach the suspect after they pass the haunted mansion

CINDERELLA'S CASTLE

- This equation gives the dimensions of the parade route around Cinderella's castle. How long will it take to make a full loop around the castle, if *t* is in minutes?
- $(x-2)^2/9 + 9(y+12)^2 = 1$

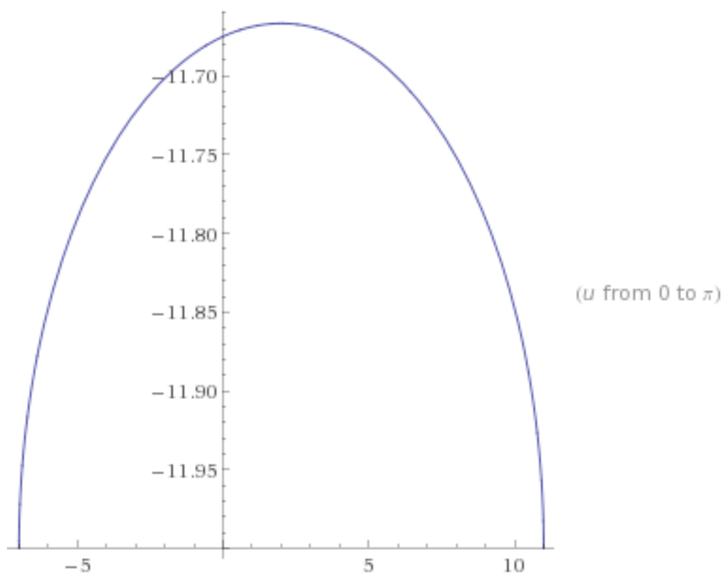
SOLUTION

- Using vectors is helpful here, since a variable *t* can be applied to get total time around the ellipse.
- First, parameterize the equation with respect to *t*. Remember that $\cos^2 t + \sin^2 t = 1$.
  - $\cos t = (x-2)/3$ and $\sin t = 3(y+12)$
  - $x = 3\cos t + 2$ and $y = 1/3 \sin t - 12$

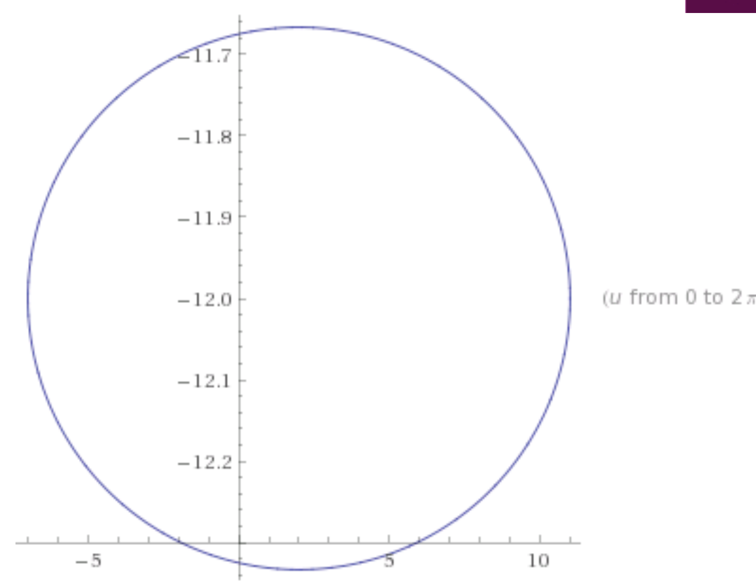
- In parametric form, this is <9cos t + 2, 1/3 sin t -12>
- Using a table of values, the position only repeats itself starting at **t = 2π.**

Please note – X and Y axes are not scaled the same.

Plotted from 0 to π



Plotted from 0 to 2π



ARIEL SWIMMING

- The position of Ariel as she swims through an underwater current is calculated by Sebastian to be <3cos(t),5sin(t)> where t represents time (in minutes). Ariel’s father constantly worries about her safety and he wants to know her velocity, speed, and acceleration when she passes Ursula’s lair at ((3sqrt(3))/2, 5/2). After t=π/2 minutes, however, he knows she will be safe due to his body guards being able to catch up with her. Being a studious advisor, Sebastian wants to write her velocity as a Cartesian equation.

SOLUTION:

- To find the vector for velocity take the first derivative of the position vector: <3cos(t), 5sin(t)> → <-3sin(t),5cos(t)>
- To find the speed find the magnitude of velocity by taking the square root of the sum of the two parts of the vector (both squared): sqrt((-3sin(t))^2+(5cos(t))^2)
- For acceleration take the second derivative of the position vector: <-3sin(t),5cos(t)> → <-3cos(t),-5sin(t)>

SOLUTION CONTINUED

- Now solve for t and plug in:

  x=3cos(t)=(3sqrt(3))/2

  y=5sin(t)=5/2

  t=π/6 (5π/6 in not in the domain)

- For the position ((3sqrt(3))/2,5/2):

  Velocity=<-3/2,(5sqrt(3))/2>

  Speed=sqrt(21)

  Acceleration=<(-3sqrt(3))/2,-5/2>

- Cartesian Equation for Velocity: y-y1=m(x-x1)
- m=y’/x’ →((5sqrt(3))/2)/(-3/2)→(5sqrt(3))/-3
- y(velocity)=(5sqrt(3)/-3)(x-(3sqrt(3)/2))+(5/2)

THOUGHTS FOR HOW VECTORS ARE RELATED TO THE WONDERFUL WORLD OF MATH

VECTORS

Calculus and Physics: Position, velocity, speed, and acceleration can be found through vectors by using limits and derivatives

Geometry: the magnitudes of vectors can often be found using the Pythagorean theorem

In general, with vectors we can better explain the world with numbers representing events happening in different dimensions

Calculus and Physics: vector functions can make finding the magnitude and direction of forces and distance (with direction) much simpler, especially when forces are being added together or combined

Works Cited

Stewart, James (2001). *Calculus* (2nd ed.). Brooks/Cole. ISBN 0-534-37718-1.

H.D. Young, R.A. Freedman (2008). *University Physics* (12th Edition ed.). Addison-Wesley (Pearson International). ISBN 0-321-50130-6.

Riley, K.F.; Hobson, M.P.; Bence, S.J. (2010). *Mathematical methods for physics and engineering*. Cambridge University Press. ISBN 978-0-521-86153-3.

Notes from Dr. Oksana Shatalov

“Calculus Questions with Answers (2)." *Calculus Questions with Answers (2)*. N.p., n.d. Web. 19 Nov. 2012. <http://www.analyzemath.com/calculus_questions/analytical/calculus_questions_2.html>. “

Hermann GraÃƒÂŸmann: Philologist