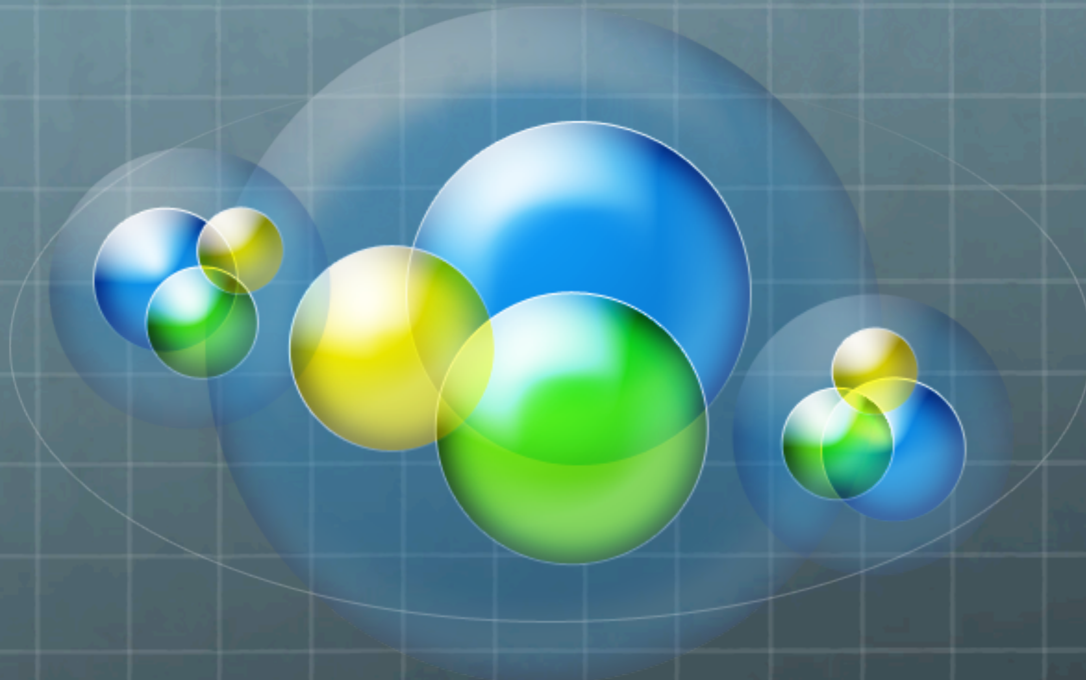
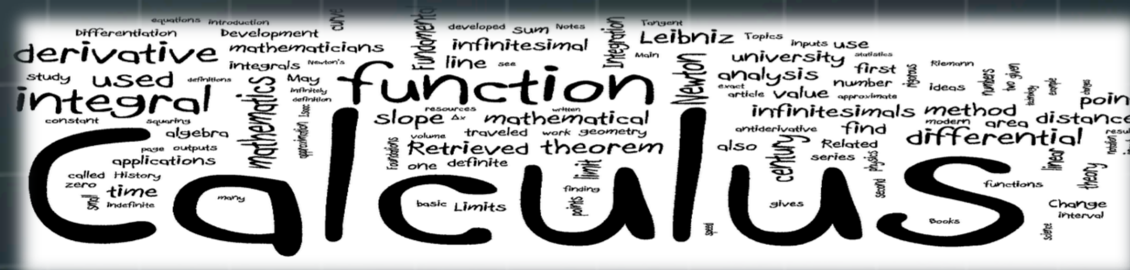


Derivatives

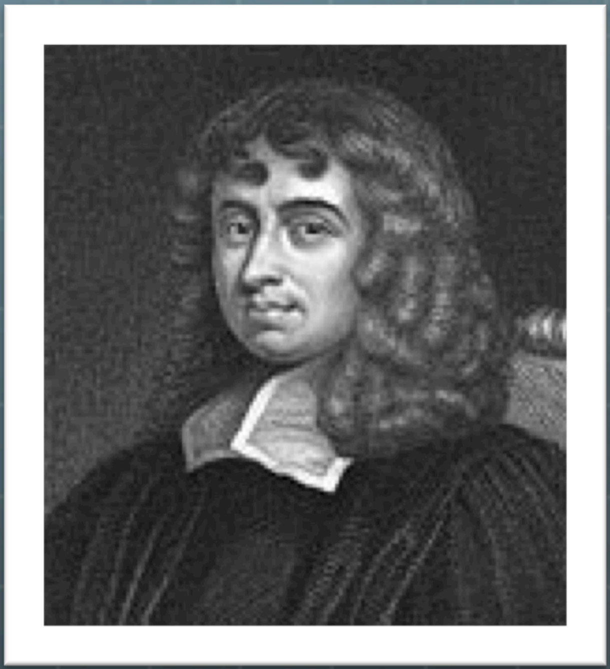
Produced By: Charles Weston Snow, Tyler Romero, Caitlyn Talbert, Alfred Stone

History

- Modern differentiation and derivatives are usually credited to Isaac Newton and Gottfried Leibniz.
- They developed the Fundamental theorem of calculus in the 17^{th} century. This related differentiation and integration in ways which revolutionized the methods for computing areas and volumes.
- However, Newton's work would not have been possible without the efforts of Isaac Barrow who began early development of the derivative in the 16^{th} century.

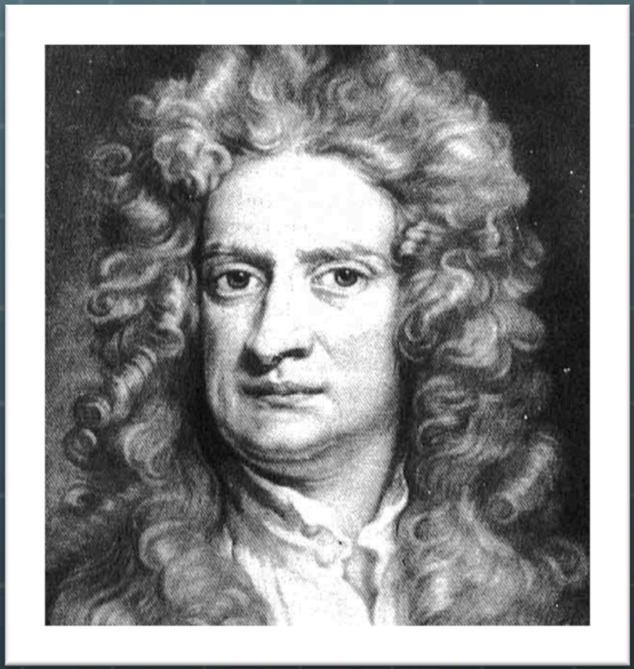


History



Isaac Barrow
(1630-1677)

Gottfried Leibniz
(1646-1716)

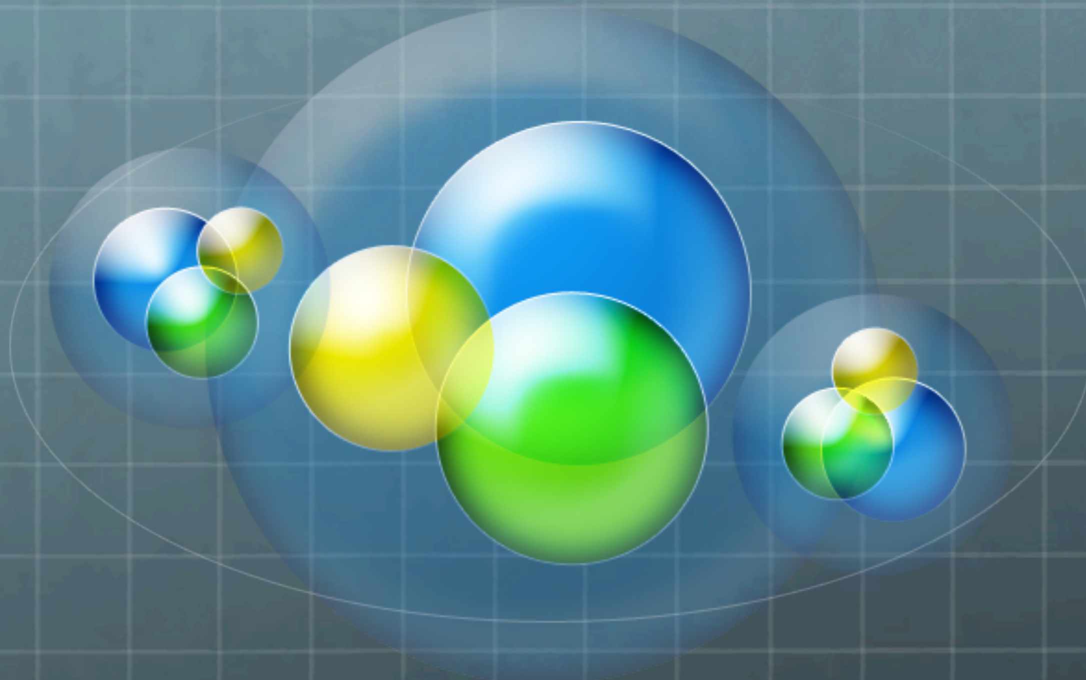


Isaac Newton
(1642-1727)

Introduction

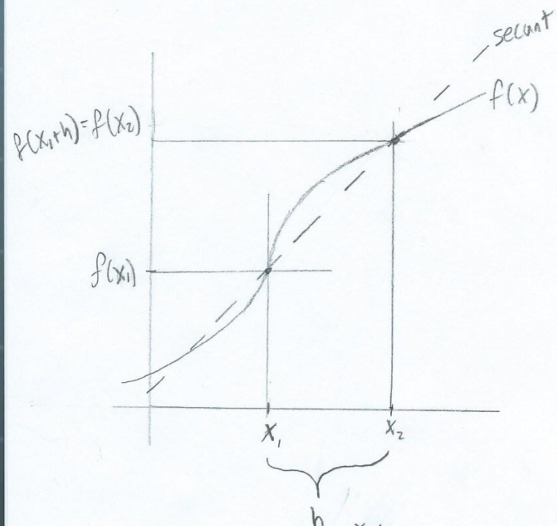
- Derivatives play a major part in our daily life.
- They are all around us.
- Have you ever been skiing?
- Have you ever driven a car?
- Have you ever walked?
- Have you ever stopped walking?
- Have you ever drawn the letter X?
- If you answered yes to any of these questions, then you could be at risk of learning. Please stay tuned for more information...

THE X FACTOR



Basic Definitions

Slopes, Secants, and Tangents



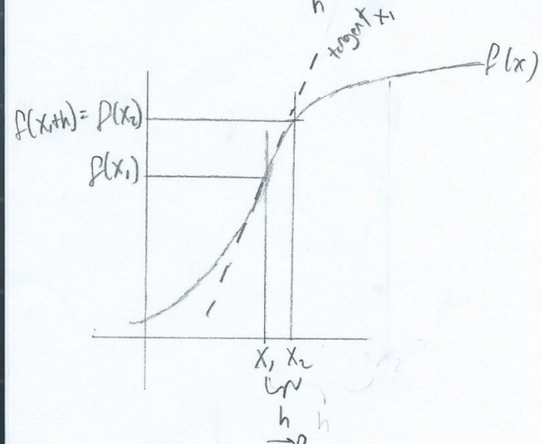
From geometry: slope $= \frac{\text{rise}}{\text{run}} = m$

$$m = \frac{\Delta y}{\Delta x}$$

$$m_{secant} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

or

$$m_{secant} = \frac{f(x_1+h) - f(x_1)}{h}$$



$$m_{tangent} = \lim_{h \to 0} \frac{f(x_1+h) - f(x_1)}{h}$$

or

$$m_{tangent} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Definition of a Derivative

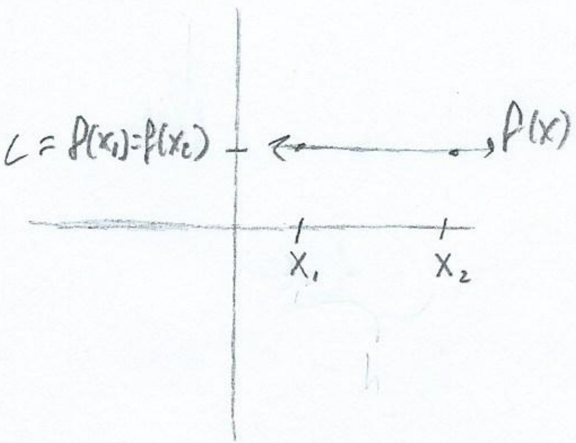
- The derivative of a function represents an infinitely small change the function with respect to one of its variables
- Another way to think of this is as the instantaneous rate of change of a function

$$f'(x) = \frac{\partial}{\partial x}\left(f(x)\right) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivation Formulas

- **For ALL derivation formulas, f'(x) and g'(x) must exist**
- **Constant Rule: if f(x)=c, then d/dx(f(x))=0**

Constant Rule



$$f(x) = c$$

$$f'(x) = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \lim_{x_2 \to x_1} \frac{c - c}{x_2 - x_1} =$$

$$= \lim_{x_2 \to x_1} \frac{0}{x_2 - x_1} = 0$$

Derivation Formulas

- **Power Rule: where n is a real number,**
  $d/dx(x^n)=nx^{n-1}$

Power Rule

$$f(x) = x^n$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} =$$

$$= \lim_{h\to 0} \frac{[(x+h)-x][(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}]}{h}$$

$$= \lim_{h\to 0} \frac{h[(x+h)^{n-1} + (x+h)^{n-2}x + \ldots + (x+h)x^{n-2} + x^{n-1}]}{h}$$

$$= x^{n-1} + x^{n-1} + \ldots + x^{n-1} + x^{n-1} = nx^{n-1}$$

$$a^1 - b^1 = (a-b)$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$a^4 - b^4 = (a-b)(a^3+a^2b+ab^2+b^3)$$

$$a^n - b^n = (a-b)(a^{n-1}+a^{n-2}b+\ldots+ab^{n-2}+b^{n-1})$$

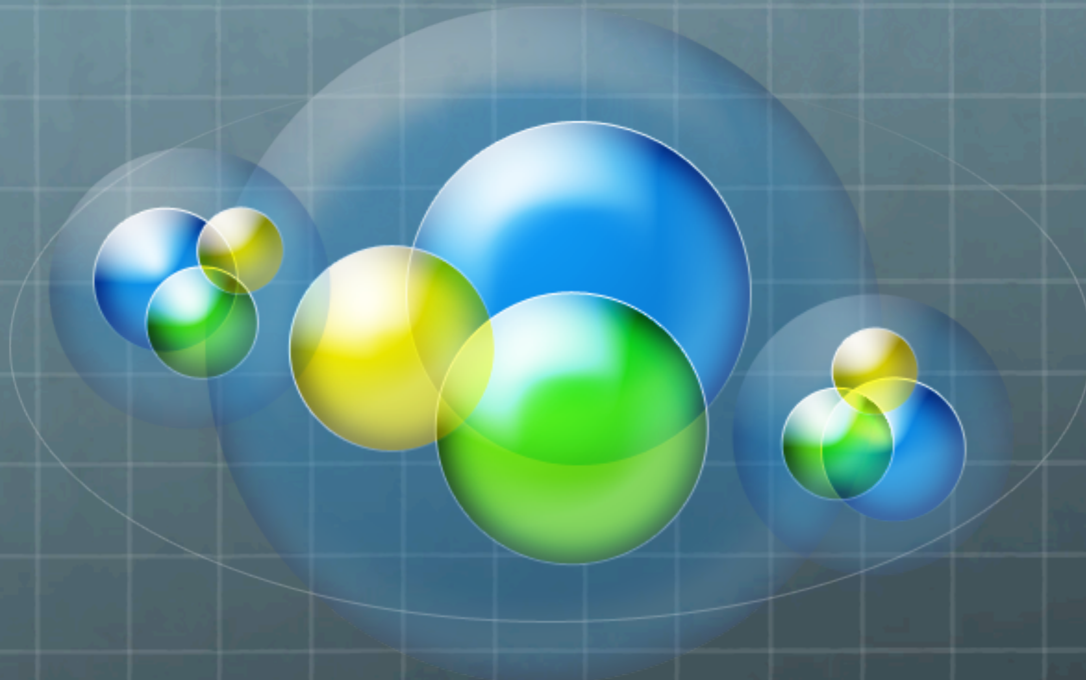
Higher Derivatives

- **Derivatives can have derivatives of their own!**

$$(f')' = f''(x) = \frac{d^2}{dx^2} f(x)$$

$$a(t) = v'(t) = x''(t)$$

a = acceleration
v = velocity
x = position



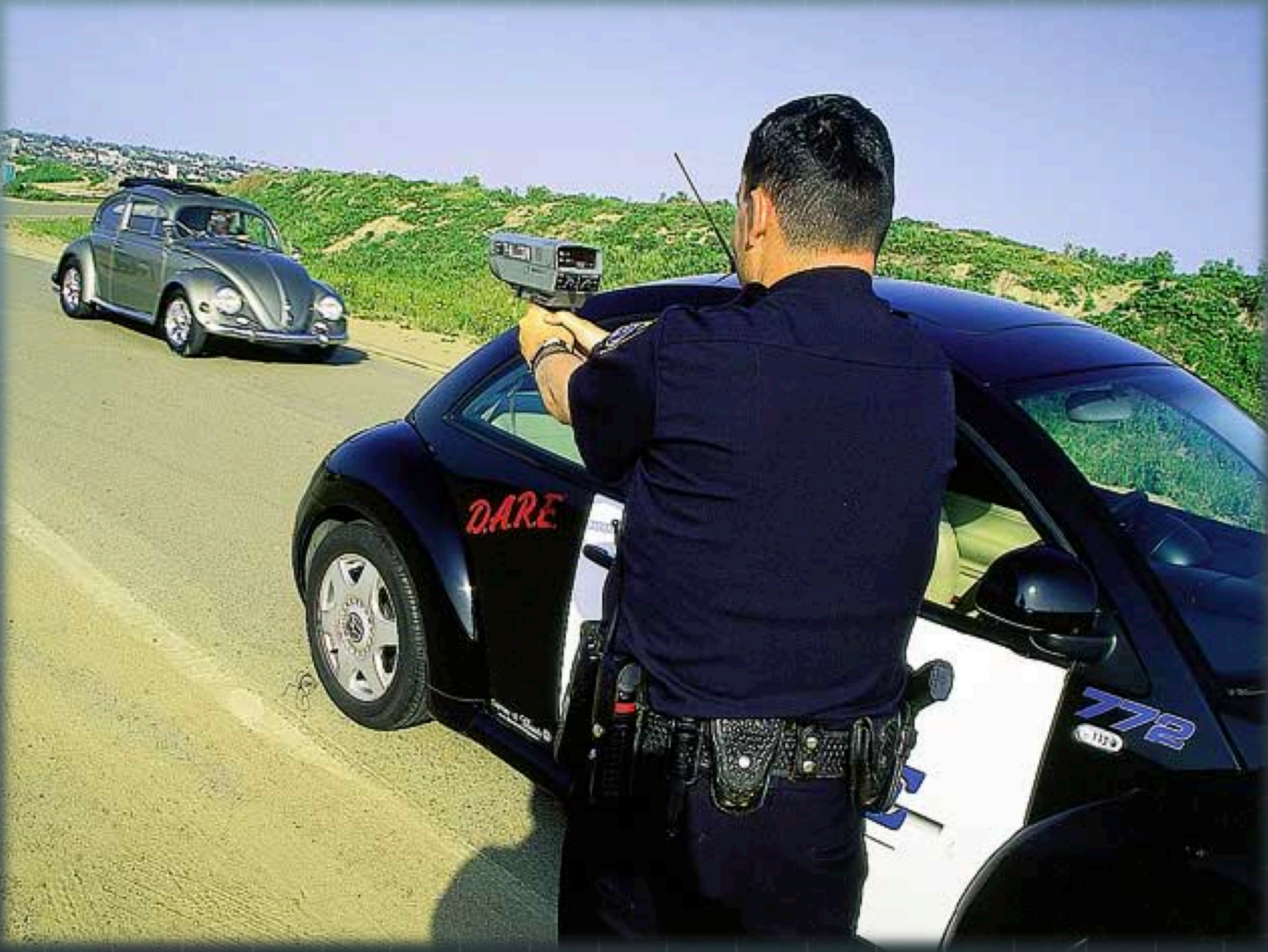
Applications

Automobile

- In an automobile there is always an odometer and a speedometer. These two gauges work in tandem and allow the driver to determine his speed and his distance that he has traveled. Electronic versions of these gauges simply use derivatives to transform the data sent to the electronic motherboard from the tires to Miles Per Hour (MPH) and distance (Km).

Radar Guns

- Keeping with the automobile theme from the previous slide, all police officers who use radar guns are actually taking advantage of the easy use of derivatives. When a radar gun is pointed and fired at your care on the highway, the gun is able to determine the time and distance at which the radar was able to hit a certain section of your vehicle. With the use of derivatives it is able to calculate the speed at which the car was going and also report the distance that the car was from the radar gun.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Business

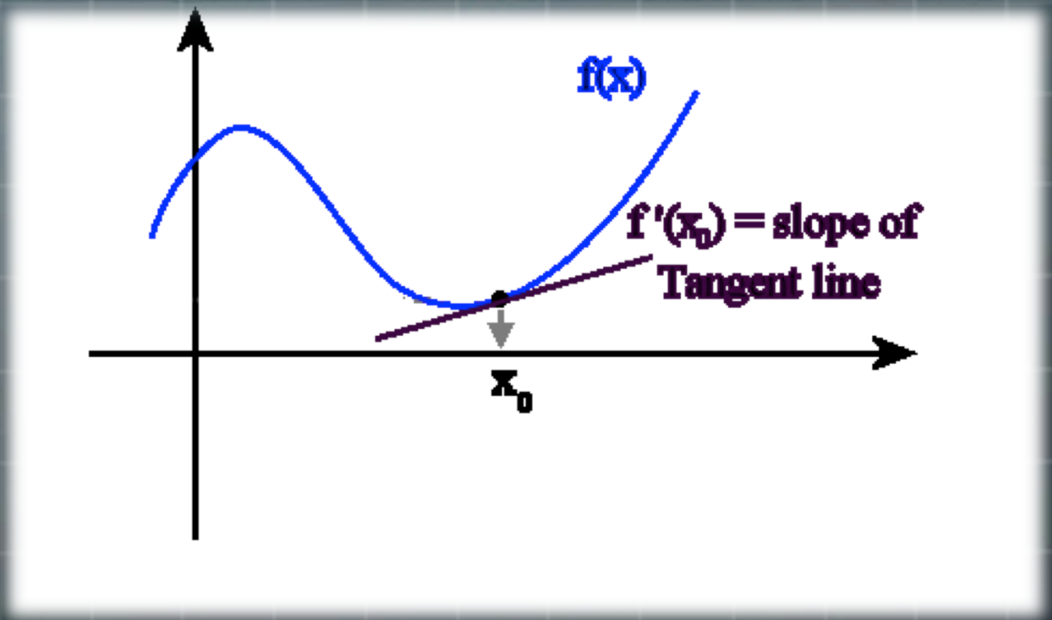
- In the business world there are many applications for derivatives. One of the most import applications is when the data has been charted on a graph or data table such as excel. Once it has be input the data can be graphed and with the application of derivatives you can estimate the profit and loss points for certain ventures.

F16

	A	B	C	D	E	F
1						
2						
3	Date	Start time	End time	Pause	Sum	Comment
4	2007-05-07	9,25	10,25	0	1	Task 1
5	2007-05-07	10,75	12,50	0	1,75	Task 1
6	2007-05-07	18,00	19,00	0	1	Task 2
7	2007-05-08	9,25	10,25	0	1	Task 2
8	2007-05-08	14,50	15,50	0	1	Task 3
9	2007-05-08	8,75	9,25	0	0,5	Task 3
10	2007-05-14	21,75	22,25	0	0,5	Task 3
11	2007-05-14	22,50	23,00	0	0,5	Task 3
12	2007-05-15	11,75	12,75	0	1	Task 3
13						
14						
15						
16						
17						

Graphs

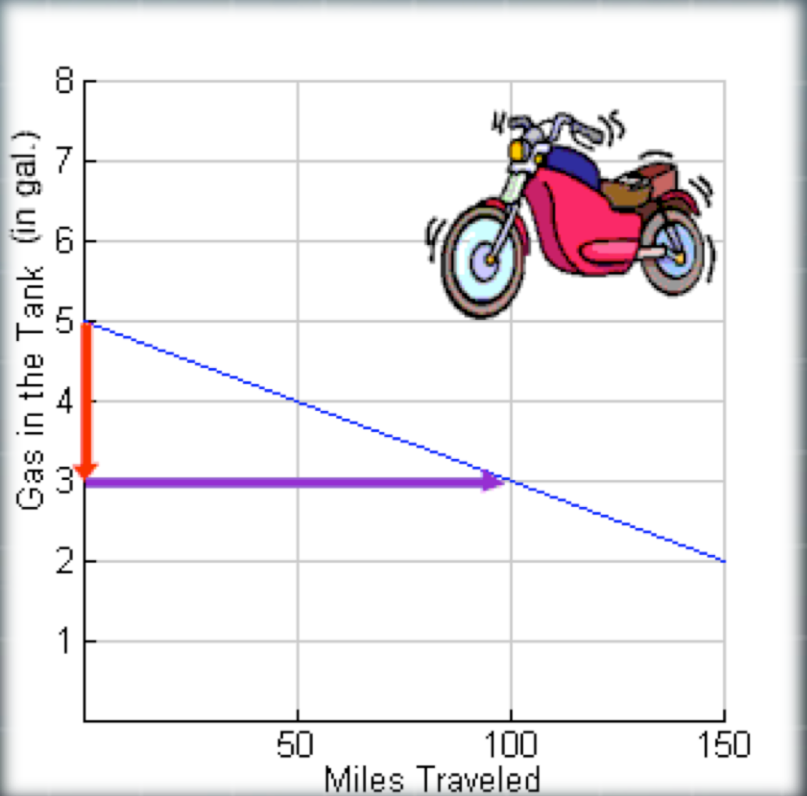
- The most common application of derivatives is to analyze graphs of data the can be collected from many different fields. Using derivatives one is able to calculate the gradient of any point of a graph.



Connecting Derivatives

Side Note:

- Derivatives can be connected to any concept involving a "rate of change." (Calculus or Not;)



Problems

Problem 1 – Circus Act

A Circus Performer is to be launched out of a cannon so that he will land in a tub of jello at the same height as the cannon, 100m away.

The Performer's horizontal and vertical positions are given by the equations:

$$x=v_{0x}t \text{ and } y=(0.5)(-9.8)t^2 + v_{0y}t$$

It is predetermined that the Performer will leave the cannon traveling at a velocity of 25m/s in the x direction.

Using the definition of a derivative, find the magnitude of velocity and magnitude of acceleration of the performer at t=3 seconds


GENERAL ADMISSION


Diagram

Problem 1 – Circus Act Solution

$x = V_{0x} t = 25t$

$100 = 25t$

$t = 4\,s$

$0 = \frac{1}{2}(-9.8)t^2 + V_{0y} t$

$0 = \frac{1}{2}(-9.8)4^2 + V_{0y}(4)$

$V_{0y} = 19.6\,m/s$

Solve for total flight time and use it to find V_{oy}

Equations

$x(t) = 25t$

$y(t) = \frac{1}{2}(-9.8)t^2 + 19.6t$

Finding V_{oy} completes the two equations

velocity Calculations

Use the definition of a derivative to find components of velocity

$$x'(t) = V_x(t) = \lim_{h \to 0} \frac{25(t+h) - 25t}{h} = 25 \lim_{h \to 0} \frac{t+h-t}{h} = 25 \lim_{h \to 0} \frac{h}{h} = 25$$

$$y'(t) = V_y(t) = \lim_{h \to 0} \frac{y(t+h) - y(t)}{h} = \lim_{h \to 0} \frac{\frac{1}{2}(-9.8)(t+h)^2 + 19.6(t+h) - \left(\frac{1}{2}(-9.8)t^2 + 19.6t\right)}{h}$$

$$= \lim_{h \to 0} \frac{-4.9(t^2 + 2th + h^2) + 19.6t + 19.6h + 4.9t^2 - 19.6t}{h}$$

$$= \lim_{h \to 0} \frac{-4.9t^2 - 9.8th - 4.9h^2 + 19.6h + 4.9t^2}{h} = \lim_{h \to 0} -9.8t - 4.9h + 19.6 = -9.8t + 19.6$$

$V_x(3) = 25$

$V_y(3) = -9.8$

$$|V| = \sqrt{V_x^2 + V_y^2} = \sqrt{25^2 + (-9.8)^2} = 26.85$$

$$\boxed{|V| = 26.85\,m/s}$$

Use the components of velocity find the magnitude of velocity

Problem 1 – Circus Act Solution

Acceleration Calculations

$V_x(t) = 25$

$V_y(t) = -9.8t + 19.6$

Velocity equations left over from previous side

$V_x'(t) = a_x(t) = \lim_{h \to 0} \frac{V_x(t+h) - V_x(t)}{h} = \lim_{h \to 0} \frac{25 - 25}{h} = 0$

$V_y'(t) = a_y(t) = \lim_{h \to 0} \frac{V_y(t+h) - V_y(t)}{h} = \lim_{h \to 0} \frac{-9.8(t+h) + 19.6 - (-9.8(t) + 19.6)}{h}$

$= \lim_{h \to 0} \frac{-9.8t - 9.8h + 19.6 + 9.8t - 19.6}{h} = \lim_{h \to 0} -9.8 = -9.8$

Use the definition of a derivative to find components of acceleration

As you can see, acceleration in projectile motion has no x-component, and its y-component is $-9.8 m/s^2$. Therefore, the magnitude of acceleration is simply $9.8 m/s^2$. In physics this value is represented by “g.”

Common Mistakes

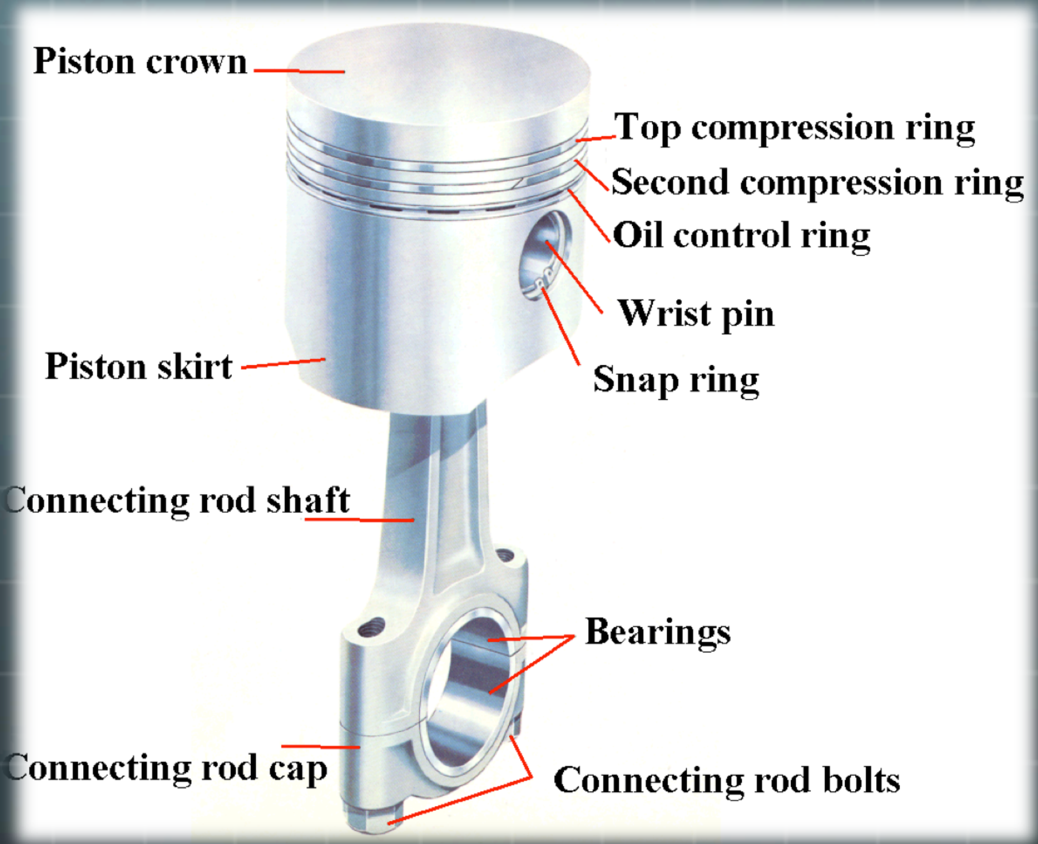
- Don’t forget that the magnitude of velocity is always positive!

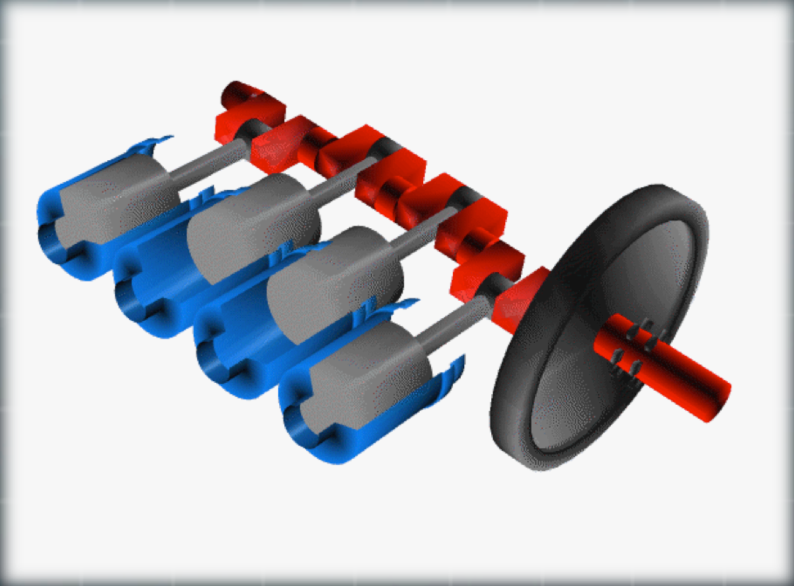
Problem 2 - Piston

A piston in the engine of James Bond's Aston Martin moves up and down according to the equation $y(t)=\sin(t)\cos(t)$, where y is in feet and t is in seconds.

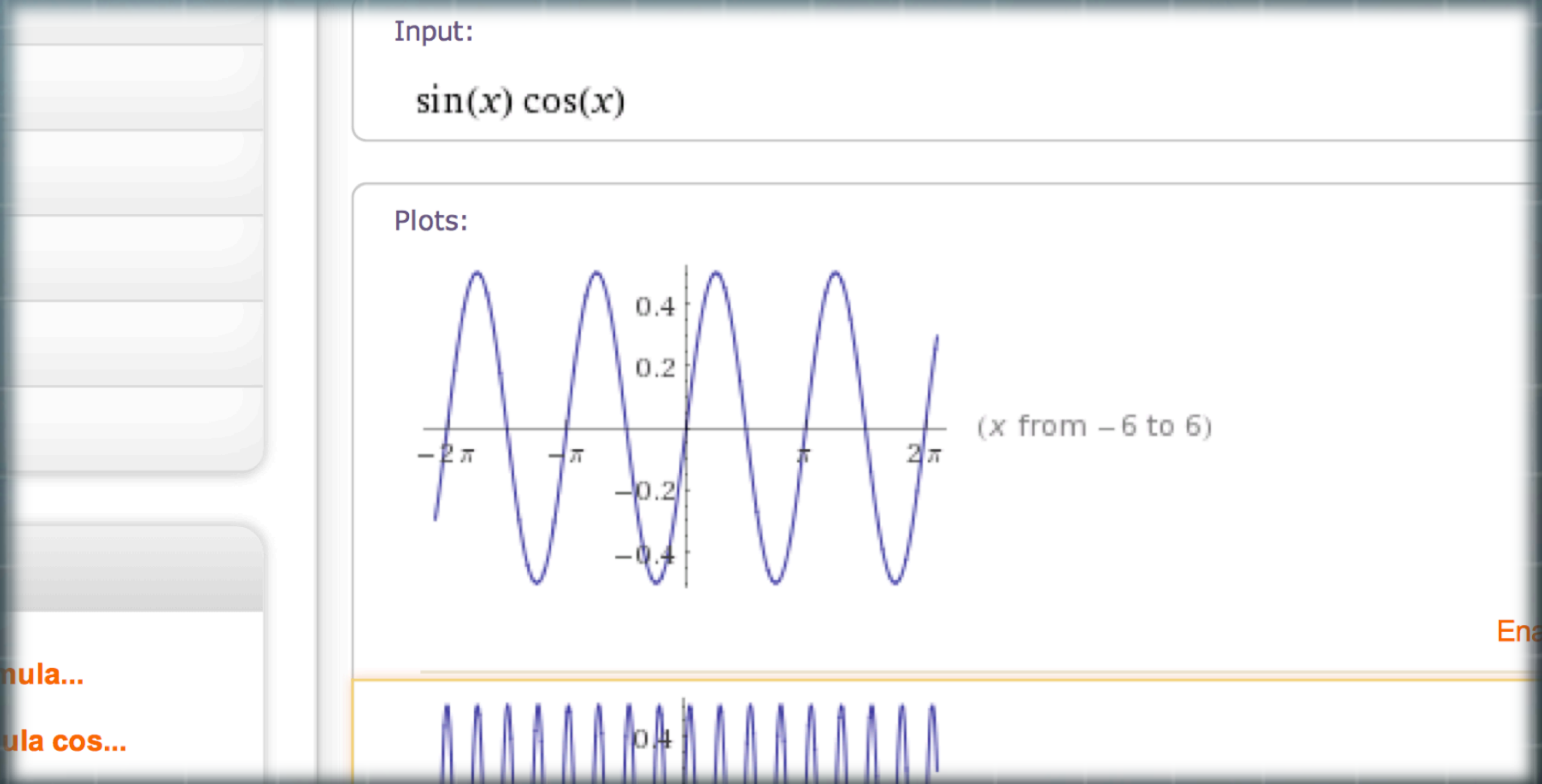
Find the amplitude and maximum velocity of the piston.

Piston Visualizations





Plotting The Function



Problem 2 – Piston Solution

Initial Equation

$$y(t) = \sin(t)\cos(t)$$

$$y'(t) = v(t) = (\sin t)'\cos t + (\cos t)'\sin t = \cos^2 t - \sin^2 t$$

☆ At amplitude, $v(t) = 0$

$$\cos^2 t - \sin^2 t = 0 \Rightarrow \cos^2 t = \sin^2 t$$

$$t = \frac{\pi}{4}(2n+1) \text{ where } n \text{ is a positive integer}$$

$$y\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

$$\boxed{A = \frac{1}{2}\ \mathrm{ft}}$$

Use differentiation formulas to find velocity as a function

Since the equation is periodic, the condition is satisfied at multiple times

Plug in any time where v(t)=0 to find the amplitude

Problem 2 – Piston Solution

$v(t) = \cos^2 t - \sin^2 t$ ← Velocity equation from previous slide

$v'(t) = a(t) = (\cos^2 t - \sin^2 t)' = (\cos^2 t)' - (\sin^2 t)'$

$a(t) = 2\cos t(-\sin t) - 2\sin t \cos t$ ← Use differentiation formulas to find acceleration equation

$= -4\cos t \sin t = -2(2\cos t \sin t)$

$a(t) = -2\sin(2t)$ ← Use trig identities to simplify

* Velocity is maximum when acceleration is zero

$-2\sin(2t) = 0 \Rightarrow \sin(2t) = 0$

let $2t = x$

$\sin x = 0 \Rightarrow x = \pi n$, where n is an integer

$2t = \pi n$

$t = \frac{\pi}{2} n$ ← Again, equation is periodic

$v(0) = \cos^2(0) - \sin^2(0) = 1^2 - 0^2 = 1$ ← Plug in any time where a(t)=0 to find maximum velocity

$V_{max} = 1 \text{ ft/s}$

Common Mistakes

- Forgetting to make the derivative of cos(x)… [-sin(x)] negative.
- Also, don't forget that it is ok to have more than one solution!

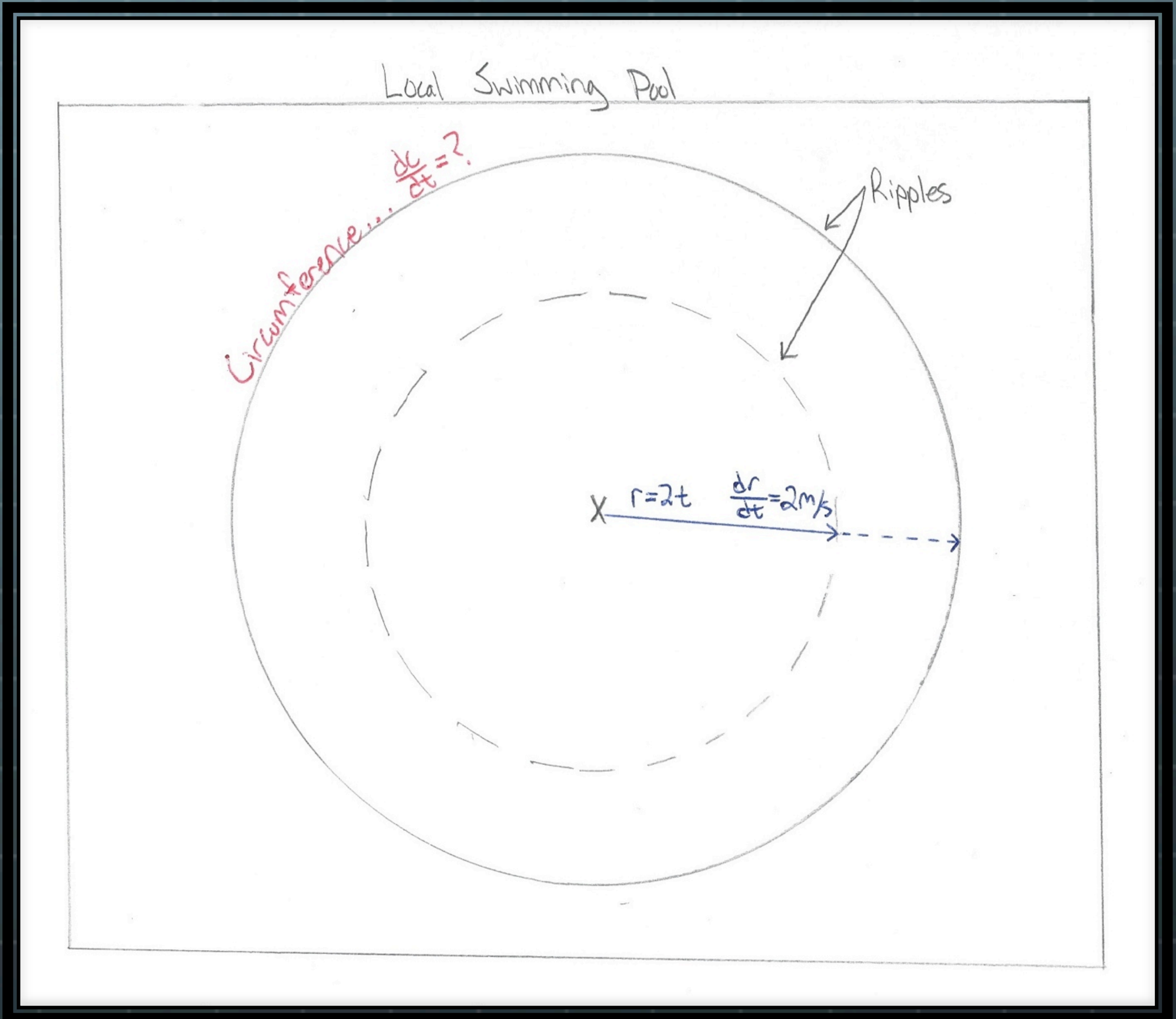
Problem 3 - Ripples

Big Bubba jumps out of a balloon from a height of 10m and lands in the local swimming pool. His splash reaches a maximum height of 7m drenching the sunbathers nearby. The ripple he generates travels outward at a rate of 2m/s.

a) Find the rate of change of the area engulfed by the ripple with respect to time.

b) Find the rate of change of the circumference of the ripple with respect to time.

Diagram



Local Swimming Pool
Circumference... dc/dt=?
Ripples
r=2t
dr/dt=2m/s

Bubba

Bubba’s Splash

Problem 3 – Ripples Solution

a)

Formula for area of a circle → $A = \pi r^2$

The ripple expands at a rate of 2m/s → $r = 2t$, $\frac{\partial r}{\partial t} = 2\,m/s$

Differentiate with respect to time → $\frac{\partial A}{\partial t} = 2\pi r \frac{\partial r}{\partial t}$

Substitute and simplify → $\frac{\partial A}{\partial t} = 2\pi(2t)(2)$

$$\boxed{\frac{\partial A}{\partial t} = 8\pi t}$$

Problem 3 – Ripples Solution

Formula for circumference of a circle

b) $C = 2\pi r$

$r = 2t$

$\frac{\partial r}{\partial t} = 2 m/s$

The ripple expands at a rate of 2m/s

Differentiate with respect to time

$$\frac{\partial C}{\partial t} = 2\pi \frac{\partial r}{\partial t}$$

$$\frac{\partial C}{\partial t} = 2\pi(2)$$

$$\boxed{\frac{\partial C}{\partial t} = 4\pi}$$

Substitute and Simplify

Common Mistakes

- Don't forget to differentiate before substituting!!!

Conclusion

- Ultimately derivatives play a vital role in our lives whether we realize it or not.
- We challenge you to look for derivatives tomorrow as you brush your teeth, sit in class, ride your bike, fly to the moon, sky dive, play a life size game of chess, or even while you're in the middle of a game of table tennis.
- Never forget: derivatives are everywhere.

Citation

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