Math 220 – Homework 11

Due Thursday 11/29 at the beginning of class

Total points: 124

PART A

Problems from the textbook:

- page 234 # 9.11 (b, e, f) [24 points], 9.12 (e) * [10 points]

PART B

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 2019$.
   (a) * [10 points] Compute $f([-3, 3])$.*(Give a formal proof.)*
   (b) * [10 points] Compute $f^{-1}([-3, 3])$.*(Give a formal proof.)*
2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^{2k}$, where k is an arbitrary natural number.
   (a) * [10 points] Compute $f([-2, 2])$.*(Give a formal proof.)*
   (b) * [8 points] Compute $f([-2, 0])$.*(Give a formal proof.)*
3. [16 points] For each of the following functions write out $f(A)$ and $f^{-1}(B)$ for the given sets A and B, where $f : \mathbb{Z} \to \mathbb{Z}$.**(No proofs are necessary.)**
   (a)
   $$f(n) = \begin{cases} 1 - n & \text{if} \quad n \in \mathbb{E} \\ 2 - n & \text{if} \quad n \in \mathbb{O} \end{cases}, \quad A = \{0, 1, 7, 11\}, \quad B = \mathbb{O}.$$
   (b) $f(n) = n^4$, $A = \{-2, -1, 0, 1, 2\}$, $B = \{2, 7, 11\}$
4. * [10 points] Disprove the statement: "For every $f : X \to Y$ and for all $A, B \subseteq X$, if $f(A) \subseteq f(B)$, then $A \subseteq B$."
5. Let $f : X \to Y$ and $A \subseteq X$.
   (a) * [10 points] Prove that $A \subseteq f^{-1}(f(A))$.
   (b) * [6 points] Disprove that $f^{-1}(f(A)) \subseteq A$.
   (c) * [10 points] Prove that if f is injective, then $f^{-1}(f(A)) = A$.