Math 220 – Homework 12

Due Thursday 4/25 at the beginning of class

Total points: 114

PART A

Problems from the textbook:

- Section 6.1

problem	1*	2*	3*
points	10	10	10

- Section 6.2

problem	1(a,b)	3*
points	30	10

PART B

1. [3 points] Let S be a nonempty subset of $\mathbb{Z}^+$. Complete the following sentence:
   "An element a is not the smallest element of S if ..."

2. * [10 points] Prove the following so called Modified form of the Principle of Mathematical Induction deriving it from PMI.

   Let $P(n)$ be a statement about the integer n so that n is a free variable in $P(n)$. Suppose that there is an integer n_0 such that

   (a) The statement $P(n_0)$ is true.

   (b) For all positive integers k such that $k \geq n_0$, if $P(k)$ is true, then $P(k+1)$ is also true.

   Then $P(n)$ is true for every positive integer $n \geq n_0$.

3. [6 points] Restate the following so called Strong Principle of Mathematical Induction in set theory language.
   (`Hint: see the proof of the Theorem 1 in notes.`)

   Let $P(n)$ be a statement about the positive integer n so that n is a free variable in $P(n)$. Suppose the following:

   (a) The statement $P(1)$ is true.

   (b) For all positive integers k, if $P(i)$ is true for every positive integer $i \leq k$, then $P(k+1)$ is true.

   Then $P(n)$ is true for every positive integer n.

4. [15 points] Let $a = -255$ and $b = 143$

   (a) Use the Euclidean Algorithm to determine $\gcd(a, b)$.

   (b) Find integers x and y such that $ax + by = \gcd(a, b)$.

5. *[10 points] Let $a, b \in \mathbb{Z}$ with a and b not both zero. Prove that if $d = \gcd(a, b)$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.