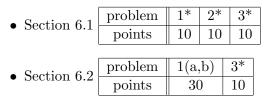
Math 220 – Homework 12

Due Thursday 4/25 at the beginning of class

Total points: 114

PART A

Problems from the textbook:



PART B

- 1. [3 points] Let S be a nonempty subset of \mathbb{Z}^+ . Complete the following sentence: "An element a is not the smallest element of S if ..."
- 2. * [10 points] Prove the following so called Modified form of the Principle of Mathematical Induction deriving it from PMI.

Let P(n) be a statement about the integer n so that n is a free variable in P(n). Suppose that there is an integer n_0 such that

- (a) The statement $P(n_0)$ is true.
- (b) For all positive integers k such that $k \ge n_0$, if P(k) is true, then P(k+1) is also true.

Then P(n) is true for every positive integer $n \ge n_0$.

3. [6 points] Restate the following so called Strong Principle of Mathematical Induction in set theory language.

(Hint: see the proof of the Theorem 1 in notes.)

Let P(n) be a statement about the positive integer n so that n is a free variable in P(n). Suppose the following:

- (a) The statement P(1) is true.
- (b) For all positive integers k, if P(i) is true for every positive integer $i \le k$, then P(k+1) is true.

Then P(n) is true for every positive integer n.

- 4. [15 points] Let a = -255 and b = 143
 - (a) Use the Euclidean Algorithm to determine gcd(a, b).
 - (b) Find integers x and y such that $ax + by = \gcd(a, b)$.
- 5. *[10 points] Let $a, b \in \mathbb{Z}$ with a and b not both zero. Prove that if $d = \gcd(a, b)$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.