Math 220 − Homework 13

Due at the beginning of Final Exam.

1. (a) Write the integer 42750 in a compact standard form.
   (b) Determine the following, representing your answer in the compact standard form:

$$\gcd(2^{2018} \cdot 3^4 \cdot 55 \cdot 7^2, 6 \cdot 3^2 \cdot 77)$$

2. Prove that if p is a prime number greater than 3, then p is of the form $3k + 1$ or $3k + 2$.

3. Prove that if p is a prime number, then $\sqrt[n]{p}$ is irrational for every integer $n \geq 2$.

4. Prove or disprove that 3 is the only prime number of the form $n^2 - 1$.

5. Prove that if a is a positive integer of the form $3n + 2$, then at least one prime divisor of a is of the form $3n + 2$.