Math 220 – Homework 1

Due Thursday 1/26 at the beginning of class

PART A

Problems from the textbook:

- Section 1.1 # 1(b,c,f,h,i,k);
- Section 1.4 # 8, 9, 10.

PART B

1. State the negation for each of the following statements.
   (a) $\sqrt{3}$ is a rational number.
   (b) 0 is not a negative number.
   (c) The real number r is at most $\sqrt{3}$
   (d) Two sides of a triangle have the same length.
   (e) The point P on the plane lies outside of the circle C.
2. For the open sentence $S(x) : 3x - 5 > 10$ over the domain $\mathbb{N}$, determine:
   (a) the values of x for which $S(x)$ is a true statement.
   (b) the values of x for which $S(x)$ is a false statement.
3. For the open sentence $P(x) : (x^2 - 16)(x^4 - 16)(x^8 + 16) = 0$ over domain D, determine:
   (a) the values of x for which $P(x)$ is a true statement if $D = \mathbb{R}$.
   (b) the values of x for which $P(x)$ is a false statement if $D = \mathbb{Z}^+$.
4. Prove that the statement $\neg((\neg Q \wedge (P \Rightarrow Q)) \Rightarrow (\neg P))$ is a tautology, a contradiction, or neither. You must state which of the three it is as well as give the proof.
5. In each of the following statements identify the hypothesis (assumption) and conclusion.
   (a) If a is irrational, then $2a$ is irrational.
   (b) a^3 is an even integer whenever a is an even integer.
6. Without changing their meanings, convert each of the following sentences into a sentence having the form "If P, then Q."
   (a) A function is integrable provided the function is continuous.
   (b) A function is rational if it is a polynomial.
   (c) "Whenever people agree with me I feel I must be wrong." (Oscar Wilde)
7. Without changing its meaning, convert the sentence *"If $xy = 0$, then $x = 0$ or $y = 0$, and conversely."* into a sentence having the form "P if and only if Q."
8. Consider the statements: $P : 2018 \in 5\mathbb{Z}$, and $Q : 5^{2018} \in \mathbb{O}$. Write each of the following statements in words and indicate whether it is true or false.
   (a) P; (b) Q; (c) $\neg P$; (d) $P \vee Q$; (e) $P \wedge Q$; (f) $P \Rightarrow Q$; (g) $\neg Q \Rightarrow P$; (e) $P \Leftrightarrow Q$.