

## Math 220 – Homework 1

Due Thursday 01/24 at the beginning of class

Total points: 236 (Writing portion 125pts) (Problems marked by \* will count toward writing portion.)

### PART A\*

Problems from the textbook:

• Section 1.1	problem	2(b)	3(b)	4(d,e)
	points	5	5	10

### PART B

- 24 points Determine whether each of the following sentences is a proposition, predicate, or neither.

(a) Harry Potter.	(e) Schreck is six feet tall.
(b) What an impossible question!	(f) All of your classmates play tennis.
(c) Multiply the function by 6.	(g) $1 + 2 + 3 + 4 + 5$
(d) $x^2 + 1 = 0$	(h) The square of every real number is positive.
- 24 points State the negation for each of the following propositions.

  - Texas A&M University has no major in Mathematics.
  - $33 - 75 < -100$
  - Today is Sunday.
  - The integer 0 is not a negative number.
  - The center of the sphere  $S$  lies inside of  $S$ .
  - At least two of my library books are overdue.
- 50 points Consider the propositions  $P : 2020^{2019} \in 4\mathbb{Z}$  and  $Q : x \notin \{a, b, c, x, y, z\}$ .

  - Write each of the following compound statements in words and indicate whether it is true or false.
    - $P$ ; (b)  $Q$ ; (c)  $\neg P$ ; (d)  $P \vee Q$ ; (e)  $\neg(P \wedge Q)$ ; (f)  $P \Rightarrow Q$ ; (g)  $\neg Q \Rightarrow P$ ; (e)  $P \Leftrightarrow Q$ .
    - converse of  $P \Rightarrow Q$ ; (g) contrapositive of  $P \Rightarrow Q$ ;
- 10 points For the predicate  $P(x) : (x^2 - 16)(x^4 - 16) = 0$ , where  $x \in \mathcal{U}$ , determine:

  - the values of  $x$  for which  $P(x)$  is a true statement if  $\mathcal{U} = \mathbb{R}$ .
  - the values of  $x$  for which  $P(x)$  is a false statement if  $\mathcal{U} = \mathbb{N}$ .
- Consider the following statement:

If the set  $A$  contains  $x$  and  $y$ , then it contains either  $a$  or  $b$ .

  - 3 points Rewrite the given statement in symbols.
  - \* 25 points For each of the following, determine whether *the given* statement is true or false. Justify your answer in each case.
 

i. $(x \notin A) \wedge (a \in A)$ ;	iv. $(x \in A) \wedge (y \in A) \wedge (a \in A) \wedge (b \in A)$ ;
ii. $(x \in A) \wedge (y \in A) \wedge (a \notin A) \wedge (b \notin A)$ ;	
iii. $(x \notin A) \wedge (a \notin A) \wedge (b \in A)$ ;	v. $(x \notin A) \wedge (y \notin A) \wedge (a \notin A) \wedge (b \notin A)$
- \* 10 points For the predicates  $p(x) : x + 1 \geq 4$  and  $q(x) : 13 < 4x$  over a domain (universe)  $S = \{0, 2, 3, 4, 6\}$ , determine all values of  $x \in S$  for which the biconditional  $P(x) \Leftrightarrow Q(x)$  is true. Show all work.

NAME (print) \_\_\_\_\_

Section (circle) 903 904

Rewrite each sentence according to provided guidelines (see “Communicating Mathematics” on eCampus), then using complete sentences give a reason to corrections you made.

1. If  $x \wedge y$  are integers of the same parity, then  $x + y$  is even.
  
2. The product of two rational numbers  $m$  and  $n$  is rational.
  
3. All solutions of the given equation are  $> 0$ .
  
4.  $x^2 + 1$  is nonnegative for every real  $x$ .
  
5. A rational number is a real number that cannot be expressed as a ratio of integers, e.g. as a fraction.
  
6. Pure mathematics topics often turn out to have applications, i.e. number theory in cryptography.
  
7. Let  $f$  and  $G$  be two given functions.
  
8. If  $n$  is even, then  $n = 2m$ .
  
9. The sets  $A$  and  $B$  contains elements  $M, N$ .