Math 220 – Homework 2

Due Wednesday 02/03 at the beginning of class

PART A

Problems from the textbook:

- Section 1.1 # 2(c,e,f,h); 3(c,e,f,h) 5(b,c,e,f).
- Section 1.2 # 5(b,c,e,f); 13c

PART B

1. Prove that the statement $\neg((\neg Q \wedge (P \Rightarrow Q)) \Rightarrow (\neg P))$ is a tautology, a contradiction, or neither. You must state which of the three it is as well as give the proof.

2. Given a quantified statement
$$\exists a \in \mathbb{Z}^+ \ni \forall b \in \mathbb{Z}^+, ab \in \mathbb{O}. \tag{1}$$
   (a) Express the given statement (1) in words.
   (b) Express the **negation** of the given statement (1) in symbols. **(Do NOT use the symbol $\notin$.)**
   (c) Express the **negation** of the given statement (1) in words.

3. Negate the following statements:
   (a) There is a cold medication that is safe and effective.
   (b) If x is a real positive number, then there is a real positive number ε such that $x < \varepsilon$ and $\frac{1}{\varepsilon} < x$.

4. Given a quantified statement
$$\forall n \in \mathbb{O},\ \exists x \in \mathbb{Z}\ \ni\ \ n = 4x + 1\ \vee\ n = 4x + 3. \tag{2}$$
   (a) Express the statement (2) in words.
   (b) Express the **negation** of the statement (2) in symbols. **(Do NOT use the symbol $\notin$.)**

5. Consider the following statement:

   "If x is a real number, then there is an integer number n such that $n \leq x < n + 1$."

   (a) Express the given statement in symbols. (**Do not use** "$\Rightarrow$")
   (b) Express the **negation** of the given statement in symbols in a useful form .

6. Consider the following definition:

   A real-valued function $f(x)$ is said to be **decreasing** *on the closed interval $[a, b]$, if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*
   (a) Write the negation of this definition.
   (b) Give an example of a decreasing function on $[-1, 1]$.
   (c) Give an example of a function that is not decreasing on $[-1, 1]$.