

## Math 220 – Homework 2

Due Thursday 1/31 at the beginning of class

Total points: 165

### PART A

Problems from the textbook:

• Section 1.1	problem	15(a,b,e, g, h,i)	16(a,b,c,d,e)
	points	30	50

### PART B

1. 20 points In each of the following statements identify the hypothesis (assumption) and conclusion. (Hint: In some cases you may express the given statement in a conditional form ( If-then), and so discover its hypothesis and conclusion.)

- In an isosceles triangle the angles at the base are equal.
- If  $x$  or  $y$  are irrational, then  $x - y$  is irrational.
- A sufficient condition for a triangle to be isosceles is that it has two equal angles.
- $a^3$  is an even integer whenever  $a$  is an even integer.
- A necessary condition for voting is that you be 18 years old.

2. 4 points Without changing its meaning, convert the sentence

*If a function has a constant derivative, then it is linear, and conversely.*

into a sentence having the form “ $P$  if and only if  $Q$ .”

3. 25 points Negate the following statements:
- Every prime number is greater than 1.
  - There are sets that contain infinitely many elements.
  - There is a cold medication that is safe and effective.
  - The number  $p$  is prime or the number  $q$  is not prime.
  - If  $f$  is a linear function, then  $f$  is continuous at 0.
4. 25 points Consider the implication “*If  $m$  and  $n$  are odd, then  $mn$  is odd.*”
- State the implication using “only if”.
  - State the converse of the implication.
  - State the contrapositive of the implication.
  - State the implication as a disjunction.
  - State the negation of the implication as a conjunction.

5. Given a quantified statement

$$\forall a, b \in \mathbb{R}, (a < b) \Rightarrow (\exists r \in \mathbb{Q} \ni (a \leq r < b)). \quad (1)$$

- 3 points Express the statement (1) in words.
- 8 points Express the **negation** of the statement (1) in symbols. (**Do NOT** use the symbol “ $\notin$ ” and interval notation.)