Math 220/970(HNR)-Homework 3

Due Wednesday 09/30 at the beginning of class

PART A

Problems from the textbook:

Section 2.1 # 1(b, c, e, i); 2(b, f,h); 4(b,c,f,i); 5; 7(e, f, h); 8(b,c, d); 13; 14; 15; 19(b, c); 20(c, f).

Section 2.2 # 4(b, e), 5(b, e), 6, 22(a); 26

PART B

1. Determine the truth or falsehood of the following statements.

    (a) The contrapositive of the statement

    "*If all elements of A are elements of B, then A is a subset of B*"

    is the statement

    "*If A is a subset of B, then all elements of A are elements of B*".

    (b) $\{a, b\} = \{b, a, b\}$

    (c) $\{x \in \mathbb{N} | -x \in \mathbb{N}\} = \emptyset$.

    (d) If $A = \{m \in \mathbb{Z} | 2 < m \leq 5\}$ then $|A| = 4$.

    (e) The empty set is a subset of every set except itself.

2. Let A, B, and C be nonempty sets. *Determine the truth or falsehood of the following statements.* (Write TRUE or FALSE for each statement.)

    (a) $A - A = \emptyset$.

    (b) $A \subset A$.

    (c) $A - B = C - B$ implies $A = C$.

    (d) If A is not a subset of B and B is not a subset of A, then $A \cap B = \emptyset$.

    (e) $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.

    (f) $A \cup A = A \cap A$ for all sets A.

3. Let $U = \{x, y, 1, 2, 3\}$ be the universal set and let $A = \{x, y, 1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3, x, y\}$. Determine the following (show all intermediate steps):

    (a) $\overline{A} \cup (B \cap C)$

    (b) $\overline{B \cup C} \cap U$

    (c) $\overline{(A \cup B) - (B \cap C)}$

4. Disprove the following statement:

    ``Let A, B, and C be nonempty subsets of a universal set U.
    Then $A \cap B = A \cap C$ implies $B = C$.''

5. Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R} | |2x + 3| \geq 19\}$ and $B = \{x \in \mathbb{R} | |x| \leq 3\}$.

    (a) Express the sets A and B using interval notation (as an interval or a union of intervals).

    (b) Determine $\overline{A} \cap \overline{B}$ as an interval or a union of intervals.

6. Prove that if $A \subseteq B$, then $A \cup C \subseteq B \cup C$.