Math 220 (HNR) – Homework 3

point split: Total points = 108

Due Thursday 02/09 at the beginning of class

PART A

Problems from the textbook:

- Section 1.1 # D5 [5 points], D6 [5 points]

PART B

1. [10 points] Let $x \in \mathbf{R}$. Prove that if $0 < x < 1$, then $x^2 - 2x + 2 \neq 0$.
2. [10 points] Let $z \in \mathbf{R}^+$. Prove that if $z^4 - 2z^2 + 2 \leq 0$, then $z^{2017} \geq 2017$.
3. [10 points] Prove that if n is an even integer, then $n^{2017} + 17(n-1)^2 - 2017$ is even.
4. [10 points] Prove that if x and y are odd integers, then $xz - yz$ is even for every integer z.
5. [15 points] Prove the following statement by direct proof:

   ``Let $n \in \mathbf{Z}$. Then n is odd if and only if $7n + 17$ is even.''

6. Consider the following statement:

   ``For all integers x and y such that $x \neq 0$, if $x|y$, then $x^{17}|y^{17}$.''

   (a) [10 points] Prove the above statement.

   (b) [3 points] Formulate the converse statement.

7. Consider the following definition:

   A real-valued function $f(x)$ is said to be **decreasing** *on the closed interval $[a, b]$, if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*

   (a) [6 points] Write the negation of this definition.

   (b) [6 points] Give an example of a decreasing function on $[-1, 1]$ and based on the above definition explain why your example is correct.

   (c) [6 points] Give an example of a function that is not decreasing on $[-1, 1]$ and based on the negation of the above definition explain why your example is correct.

8. For each of the following statements, give an example of the domain D in which the statement is true and an example of the domain D in which the statement is false. Explain why your answers are correct.

   (a) [6 points] $\forall x \in D, ((0 < x^2 < 2) \Rightarrow (x = 1))$

   (b) [6 points] $\forall x \in D, ((0 < x^2 < 2) \Rightarrow ((x = 1) \vee (x = -1)))$