

## Math 220-Homework 3

Due Thursday 02/12 at the beginning of class

### PART A

Problems from the textbook:

- Section 1.4 # 5, 16, 17<sup>1</sup>, 20, 21

### PART B

1. Determine the truth or falsehood of the following statements. (Write TRUE or FALSE for each statement.)

- (a)  $P \Rightarrow P$  is a tautology.
- (b)  $P \Rightarrow \neg P$  is a contradiction.
- (c) The contrapositive of the statement  
*“If all elements of  $A$  are elements of  $B$ , then  $A$  is a subset of  $B$ ”*  
 is the statement  
*“If  $A$  is a subset of  $B$ , then all elements of  $A$  are elements of  $B$ ”.*
- (d)  $\{a, b\} = \{b, a, b\}$
- (e)  $\{x \in \mathbb{N} \mid -x \in \mathbb{N}\} = \emptyset$ .
- (f) If  $A = \{m \in \mathbb{Z} \mid 2 < m \leq 5\}$  then  $|A| = 4$ .

2. Given a quantified statement

$$\exists a \in \mathbb{Z}^+ \ni \forall b \in \mathbb{Z}^+, ab \in \mathbb{O}. \quad (1)$$

- (a) Express the given statement (1) in words.
  - (b) Express the **negation** of the given statement (1) in symbols. (**Do NOT use the symbol  $\notin$ .**)
  - (c) Express the **negation** of the given statement (1) in words.
3. Consider the following statement:

$$\text{“If } \sqrt{3} < \sqrt{7}, \text{ then } 3 < 7.\text{”}$$

Write in a useful form

- (a) the converse;
  - (b) the contrapositive;
  - (c) the converse of contrapositive;
  - (d) the contrapositive of converse.
4. Prove the following statement:  
 ‘‘Let  $n \in \mathbf{Z}$ . Then  $n$  is odd if and only if  $11n - 7$  is even.’’
  5. Prove the statement ‘‘If  $n$  is an even integer, then  $5n + 11$  is odd.’’ by
    - (a) a direct proof;
    - (b) a proof by contrapositive;
    - (c) a proof by contradiction.

---

<sup>1</sup>Hint: see Proposition 37 in the Lecture Notes