Math 220 – Homework 3

Due Thursday 2/07 at the beginning of class

Total points: 164 (Writing portion 140 pts) (Problems marked by $*$ will count toward writing portion.)

1. [6 **points**] Consider the following definition:

   A real-valued function $f(x)$ is said to be **one to one** *if for all $x_1, x_2 \in \mathbb{R}$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.*

   Write the negation of this definition completing the following: *"A real-valued function $f(x)$ is said to be* **not one to one** *if ..."*

2. Consider the following definition:

   A real-valued function $f(x)$ is said to be **decreasing** *on the closed interval $[a, b]$, if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*

   (a) [6 **points**] Write the negation of this definition completing the following: *"A real-valued function $f(x)$ is said to be* **not decreasing** *on the closed interval $[a, b]$, if ..."*

   (b) [6 **points**] Give an example of a function that is decreasing on the interval $[1, 3]$ and based on the above definition explain why your example is correct.

   (c) [6 **points**] Give an example of a function that is not decreasing on the interval $[1, 3]$ and based on the negation of the above definition explain why your example is correct.

3. * [10 **points**] Let $x, y, z \in \mathbf{R}$. Prove that if $\sin(x^2 + 2y^4 + 3z^6) < \frac{1}{3}$, then $x^2 + 2y^4 + 3z^6 + \frac{1}{3} \neq 0$.

4. * [10 **points**] Let $x \in \mathbf{R}^+$. Prove that if $\sin(x + 3) \geq 3$, then $x^{2019} \geq 2019$.

5. * [10 **points**] Prove that the product of every two odd integers is odd. (Give a formal proof.)

6. * [10 **points**] Prove that the sum of every three consecutive integers is divisible by 3. (Give a formal proof.)

7. * [10 **points**] Prove that for every natural number n, $20n^2 - 6^n - 3$ is odd. (Give a formal proof. Don't use proof by cases!)

8. *[10 **points**] Prove that for every integer x, the integers $13x - 11$ and $17x + 2$ are of opposite parity.

9. * [30 **points**] Let a and b be integers. Give the formal proof for following statements.

   (a) If $a|b$, then $a|(b^4 - b^2 + 5b)$.

   (b) For every $n \in \mathbb{N}$, if $a|b^n$, then $a|(b^{3n} + b^{2n} - b^n)$.

   (c) If $a^2|a$, then $a \in \{-1, 0, 1\}$.

10. * [50 **points**] Prove or disprove the following statements:

    (a) For every integer n, if n is divisible by 2 and n is divisible by 6, then n is divisible by 12.

    (b) There exists an odd integer n such that $n^2 + 2n + 3$ is odd.

    (c) If $x, y \in \mathbb{R}$, then $|x - y| = |x| - |y|$.

    (d) Let $a, b \in \mathbb{Z}$. If $a|b$ and $b|a$, then $a = b$.

    (e) There exists an even number n such that n^2 is odd.