Math 220/970(HNR)-Homework 4

Due Wednesday 10/07 at the beginning of class

PART A

Problems from the textbook:

Section 2.2 # 16(a,b); 23

Section 2.3 # 2, 4, 5(b,c,e,f), 11, 14, 23.

PART B

1. Determine the truth or falsehood of the following statements. (Write TRUE or FALSE for each statement.)
   (a) $A \times B = B \times A$ for all nonempty sets A and B.
   (b) If A is not a subset of B and B is not a subset of A, then $A \cap B = \emptyset$.
   (c) For all sets A, B, and C, $A \cup (B \cap C) = (A \cap C) \cup (B \cap C)$.
   (d) $7 \notin \{\{-1, 7\}, \{-7, 2015, 0\}, \{1, 2\}\}$.
   (e) $A \cup A = A \cap A$ for all sets A.
   (f) If $A = \{a, \{a, b, c\}\}$ and $B = \{\{c, d\}, \{a, b, c, d\}\}$ then $|A| = |B|$. .
   (g) If $\{1\} \in P(A)$, then $1 \in A$ and $\{1\} \notin A$.
2. For the sets $A = \{a, b\}$ and $B = \{0, 1\}$ form the following Cartesian products:
   (a) $B \times A$
   (b) $B \times A \times B$.
3. Let $A = \{a, b, c\}$.
   (a) Write out all the different partitions of the set A.
   (b) Write out the power set, $P(A)$, for A.
4. Give an example of two different partitions of the set $\{x | x \text{ is an integer}\}$.
5. For each $n \in \mathbb{Z}^+$, define $A_n = \{n, 2n\}$. Let $I = \{1, 2, 4\}$. Find $\bigcup_{\alpha \in I} A_\alpha$.
6. For each $n \in \mathbb{Z}^+$, define $A_n = \left\{ x \in \mathbb{R} | \ -\frac{1}{n} \leq x \leq 2 - \frac{1}{n} \right\}$. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i \in \mathbb{Z}^+} A_i$.
7. Let A, B, and C be nonempty sets. Prove the following statements.
   (a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
   (b) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
   (c) $A \times (B - C) = (A \times B) - (A \times C)$. (Hint:$(x \in A) \wedge (y \notin B) \Rightarrow ((x, y) \notin A \times B.)$)
8. Find $|A|$, where $A = \{(x, y) \in \mathbf{Z} \times \mathbf{Z} | \ |x| + |y| = 3\}$.