## Math 220 (HNR) - Homework 4

## Due Thursday 02/16 at the beginning of class

Total points $=161$

## PART A

Problems from the textbook:

- Section 1.3 \# 4 [22 points], 8 [22 points]
- Section $1.4 \# 5$ [ 6 points], 7 [ 6 points], $20[15$ points], 23 [5 points]


## PART B

1. [16 points] For the statement "If $\sqrt{3}<\sqrt{7}$, then $3<7$." write in a useful form
(a) the converse;
(b) the contrapositive;
(c) the converse of contrapositive;
(d) the contrapositive of converse.
2. [12 points] For the statement
''For every integer $n$, if $n$ is divisible by 2 and $n$ is divisible by 3 , then $n$ is divisible by 6.' ' write in a useful form
(a) the converse;
(b) the contrapositive.
3. [10 points] Prove that if $n \in \mathbf{Z}$, then $n^{3}+n$ is even.
4. [10 points] Prove that $5 x-3 y$ is even, then $x$ and $y$ are of the same parity.
5. (a) [10 points] Let $n \in \mathbf{Z}$. Prove that if $n^{2}-5 \in \mathbb{E}$, then $n^{2}-5 \in 4 \mathbb{Z}$.
(b) [5 points] Give an example of an integer $n$ such that $n^{2}-5 \in \mathbb{E}$, but $n^{2}-5 \notin 8 \mathbb{Z}$.
6. [10 points] Prove that for all integers $m$ and $n$, if the product $m n$ is even, then $m$ is even or $n$ is even.
7. [12 points] Prove or disprove the following statements:
(a) For all positive integers $x, y, z, x^{y^{z}}=\left(x^{y}\right)^{z}$.
(b) If $n \in\{0,1,2,3,4\}$, then $2^{n}+3^{n}+n(n-1)(n-2)$ is prime.
(c) For every integer $n$, if $n$ is divisible by 2 and $n$ is divisible by 6 , then $n$ is divisible by 12 .
