Math 220 (HNR) – Homework 4

Due Thursday 02/16 at the beginning of class

Total points = 161

PART A

Problems from the textbook:

- Section 1.3 # 4 [22 points], 8 [22 points]
- Section 1.4 # 5 [6 points], 7 [6 points], 20[15 points], 23 [5 points]

PART B

- 1. [16 points] For the statement "If $\sqrt{3} < \sqrt{7}$, then 3 < 7." write in a useful form
 - (a) the converse;
 - (b) the contrapositive;
 - (c) the converse of contrapositive;
 - (d) the contrapositive of converse.
- 2. [12 points] For the statement

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''For every integer n, if n is divisible by 2 and n is divisible by 3, then n is divisible by 6. ', write in a useful form
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- (a) the converse;
- (b) the contrapositive.
- 3. [10 points] Prove that if $n \in \mathbb{Z}$, then $n^3 + n$ is even.
- 4. [10 points] Prove that 5x 3y is even, then x and y are of the same parity.
- 5. (a) [10 points] Let $n \in \mathbb{Z}$. Prove that if $n^2 5 \in \mathbb{E}$, then $n^2 5 \in 4\mathbb{Z}$.
 - (b) [5 points] Give an example of an integer n such that $n^2 5 \in \mathbb{E}$, but $n^2 5 \notin 8\mathbb{Z}$.
- 6. [10 points] Prove that for all integers m and n, if the product mn is even, then m is even or n is even.
- 7. [12 points] Prove or disprove the following statements:
 - (a) For all positive integers $x, y, z, x^{y^z} = (x^y)^z$.
 - (b) If $n \in \{0, 1, 2, 3, 4\}$, then $2^n + 3^n + n(n-1)(n-2)$ is prime.
 - (c) For every integer n, if n is divisible by 2 and n is divisible by 6, then n is divisible by 12.